Algebra and Discrete Mathematics
Volume 11 **(2011)**. Number 1. pp. 23 – 45

RESEARCH ARTICLE

On partial skew Armendariz rings

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Communicated by V. V. Kirichenko

Abstract. In this paper we consider rings R with a partial action α of an infinite cyclic group G on R. We introduce the concept of partial skew Armendariz rings and partial α-rigid rings. We show that partial α-rigid rings are partial skew Armendariz rings and we give necessary and sufficient conditions for R to be a partial skew Armendariz ring. We study the transfer of Baer property, a.c.c. on right annhilators property, right p.p. property and right zip property between R and $R[x;\alpha]$.

We also show that $R[x;\alpha]$ and $R\langle x;\alpha\rangle$ are not necessarily associative rings when R satisfies the concepts mentioned above.

1. Introduction

Partial actions of groups have been introduced in the theory of operator algebras giving powerful tools of their study (see [15] and [17] and the literature quoted therein). Also in [15] the authors introduced partial actions on algebras in a pure algebraic context. Let G be a group and R a unital k-algebra, where k is a commutative ring. A *partial action* α of G on R is a collection of ideals S_g of R, $g \in G$, and isomorphisms of (non-necessarily unital) k-algebras $\alpha_g : S_{g^{-1}} \to S_g$ such that:

(i) $S_1 = R$ and α_1 is the identity map of R;

(ii) $S_{(gh)^{-1}} \supseteq \alpha_h^{-1}(S_h \cap S_{g^{-1}})$;

(iii) $\alpha_g \circ \alpha_h(x) = \alpha_{gh}(x)$, for every $x \in \alpha_h^{-1}(S_h \cap S_{g^{-1}})$.

2000 Mathematics Subject Classification: *16S36; 16S35.*

Key words and phrases: *partial actions, Armendariz rings, Baer rings and P.P rings.*

The property (ii) easily implies that $\alpha_g(S_{g^{-1}} \cap S_h) = S_g \cap S_{gh}$, for all $g, h \in G$. Also $\alpha_{g^{-1}} = \alpha_g^{-1}$, for every $g \in G$.

Given a partial action α of a group G on R, an enveloping action is an algebra T together with a global action $\beta = \{\sigma_g \mid g \in G\}$ of G on T, where σ_g is an automorphism of T, such that the partial action is given by restriction of the global action ([15], Definition 4.2). From ([15], Theorem 4.5) we know that a partial action α has an enveloping action if and only if all the ideals S_g are unital algebras, i.e., S_g is generated by a central idempotent of R, for every $g \in G$. In this case the partial skew group ring $R \star_\alpha G$ is an associative algebra (this is not true in general, see ([15], Example 3.5)).

When α has an enveloping action (T, β) we may consider that R is an ideal of T and the following properties hold:

(i) the subalgebra of T generated by $\bigcup_{g\in G} \sigma_g(R)$ coincides with T and we have $T = \sum_{g\in G} \sigma_g(R)$;

(ii) $S_g = R \cap \sigma_g(R)$, for every $g \in G$;

(iii) $\alpha_g(x) = \sigma_g(x)$, for every $g \in G$ and $x \in S_g^{-1}$.

The authors in [13] introduced the concepts of partial skew Laurent polynomial rings and partial skew polynomial rings. In these cases, the authors studied prime and maximal ideals with the assumption that the partial action has an enveloping action. Moreover, the authors in [14] studied Goldie property in partial skew polynomial rings and partial skew Laurent polynomial rings.

Let R be an associative ring with identity element 1_R, G an infinite cyclic group generated by σ and $\{\alpha_{\sigma^i} : S_{\sigma^{-i}} \to S_{\sigma^i}, i \in \mathbb{Z}\}$ a partial action of G on R. We simplify the notation by putting $\alpha_i = \alpha_{\sigma^i}$ and $S_i = S_{\sigma^i}$, for every $i \in \mathbb{Z}$. Then the partial skew group ring can be identified with the set of all finite formal sums $\sum_{i=-n}^{m} a_i x^i$, $a_i \in S_i$ for every $-n \leq i \leq m$, where the sum is usual and the multiplication rule is $ax^i bx^j = \alpha_i(\alpha_{-i}(a)b)x^{i+j}$. We denote this ring by $R\langle x; \alpha\rangle$ and call it partial skew Laurent polynomial ring. The partial skew polynomial ring $R[x; \alpha]$ is defined as the subring of $R\langle x; \alpha\rangle$ whose elements are the polynomials $\sum_{i=0}^{n} b_i x^i$, $b_i \in S_i$ for every $0 \leq i \leq n$ with usual sum of polynomials and multiplication rule as before.

In this paper, we assume that R is an associative ring and α is a partial action of an infinite cyclic group G on R such that $R[x; \alpha]$ and $R\langle x; \alpha\rangle$ are not necessarily associative rings.

In the Section 2, we study McCoy's result in partial skew Laurent polynomial rings when a partial action α of an infinite cyclic group G on R has an enveloping action (T, σ), where σ is an automorphism of T.

In the Section 3, we introduce the concept of partial skew Armendariz rings and we show that R is a partial skew Armendariz ring if and only if the canonical map from the set of right annihilators of R to the set of right annihilators of $R[x;\alpha]$ is bijective and with this result we study the transfer of Baer property , p.p-property, ascending chain condition on right annihilators property and right zip property between R and $R[x;\alpha]$. We study necessary and sufficient conditions for $R[x;\alpha]$ to be reduced and as a consequence of this result we introduce the concept of partial α-rigid rings. We show that partial α-rigid rings are partial skew Armendariz rings.

In the Section 4, we give examples to show that Baer rings, p.q. Baer rings, p.p-rings and quasi-Baer rings do not imply the associativity of the partial skew polynomial rings and partial skew Laurent polynomial rings. When a partial action α of a group G on R has an enveloping action (T,β), we study conditions to show the following equivalences: R is Baer $\Leftrightarrow$ T is Baer, R is p.q. Baer $\Leftrightarrow$ T is p.q. Baer, R is quasi-Baer $\Leftrightarrow$ T is quasi-Baer and R is p.p $\Leftrightarrow$ T is p.p. When (R,α) has an enveloping action (T,σ), where σ is an automorphism of T, we show that T is σ-rigid $\Rightarrow$ R is partial α-rigid and T is skew Armendariz $\Rightarrow$ R is partial skew Armendariz. Moreover, we give an example to show that the converse of each arrow is not true.

Throughout this article, for a non-empty subset Y of a ring S, we denote $r_S(Y)=\{a\in S : Ya=0\}$ $(l_R(Y)=\{a\in S : aY=0\})$ the right (left) annihilator of Y in S.

2. Generalization of McCoy's result in partial skew Laurent polynomial rings

In [33], McCoy proved that if R is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$ there exists a non-zero element $c\in R$ such that $cg(x)=0$, and in [26] it was proved that if $r_{R[x]}(f(x)R[x])\neq(0)$, then $r_{R[x]}(f(x)R[x])\cap R\neq(0)$, where $f(x)\in R[x]$ and $r_{R[x]}(f(x)R[x])=\{h(x)\in R[x] : f(x)R[x]h(x)=0\}$. Moreover, the author in ([12], Theorems 2.3 and 2.4), generalized these results for skew polynomial rings of automorphism and derivation type. We study this situation in partial skew Laurent polynomial rings but we are unable to prove an analogue result for partial skew polynomial rings.

Throughout this section we assume that R is not necessarily a commutative ring and α is a partial action of an infinite cyclic group G on R with enveloping action (T,σ), where σ is an automorphism of T.

Lemma 1. *Let* $f(x)=\sum_{i=p}^{n} b_i x^i$ *and* $g(x)=\sum_{i=q}^{m} a_i x^i$ *be two elements of* $R\langle x;\alpha\rangle$. *Then* $f(x)R\sum_{i=q}^{m}\alpha_j(a_i 1_{-j})x^{i+j}=0$, *for every* $j\in\mathbb{Z}$, *if and only if* $f(x)R\langle x;\alpha\rangle g(x)=0$.

Proof. Suppose that $f(x)R\sum_{i=q}^{m}\alpha_j(a_i 1_{-j})x^{i+j} = 0$, for every $j\in\mathbb{Z}$. Then, we have that

$$f(x)rx^j g(x)=f(x)r1_j x^j g(x)=f(x)r\sum_{i=q}^{m}\alpha_j(a_i 1_{-j})x^{i+j}=0.$$

So, $f(x)R\langle x;\alpha\rangle g(x)=0$.

The converse is trivial. □

Let $h(x)=\sum_{i=s}^{n} a_i x^i \in R\langle x;\alpha\rangle$. The length of $h(x)$ is the number $len(h(x))=n-s+1$ and we use this number below.

Theorem 1. *Let* $f(x)\in R\langle x;\alpha\rangle$. *If* $r_{R\langle x;\alpha\rangle}(f(x)R\langle x;\alpha\rangle)\neq 0$ *then* $r_{R\langle x;\alpha\rangle}(f(x)R\langle x;\alpha\rangle)\cap R\neq 0$, *where* $r_{R\langle x;\alpha\rangle}(f(x)R\langle x;\alpha\rangle)=\{h(x)\in R\langle x;\alpha\rangle : f(x)R\langle x;\alpha\rangle h(x)=0\}$.

Proof. We freely use Lemma 1 without mention. Let $f(x)=a_p x^p+....+a_q x^q$. If either $f(x)$ is constant or $f(x)=0$ or $f(x)=a_q x^q$, then the assertion is clear. So, assume that $q\neq 0$, $p<q$ and

$$r_{R\langle x;\alpha\rangle}(f(x)R\langle x;\alpha\rangle)\cap R=(0).$$

Let $g(x)=b_t x^t+...+b_m x^m\in r_{R\langle x;\alpha\rangle}(f(x)R\langle x;\alpha\rangle)$ of minimal length with $b_m\neq 0$. Then

$$f(x)R\langle x;\alpha\rangle g(x)=0,$$

and we have that $f(x)1_{-q}x^{-q}R\sum_{i=t}^{m}\alpha_j(b_i 1_{-j})x^{i+j}=0$, for every $j\in\mathbb{Z}$. Thus, $a_q R\alpha_j(1_{-j}b_m)=0$, for every $j\in\mathbb{Z}$. Hence, $a_q R\langle x;\alpha\rangle b_m=0$ and we have that

$$a_q R\langle x;\alpha\rangle g(x)=a_q R\langle x;\alpha\rangle(b_t x^t+...+b_{m-1}x^{m-1}).$$

So,

$$\begin{aligned} &f(x)R\langle x;\alpha\rangle a_q R\langle x;\alpha\rangle(b_t x^t+...+b_{m-1}x^{m-1}))=\\ &=f(x)(R\langle x;\alpha\rangle a_q R\langle x;\alpha\rangle)g(x)=0,\end{aligned}$$

for every $j\in\mathbb{Z}$, and we obtain that

$$f(x)R\langle x;\alpha\rangle a_q R(\alpha_j(b_t 1_{-j})x^{t+j}+...+\alpha_j(b_{m-1}1_{-j})x^{m-1+j})=0,$$

for every $j \in \mathbb{Z}$. Note that by choice of $g(x)$, we have that $a_qR(\alpha_j(b_t1_{-j})x^{t+j} + ... + \alpha_j(b_{m-1}1_{-j})x^{m-1+j})) = 0$. Thus,

$$a_qR\langle x;\alpha\rangle(\alpha_j(b_t1_{-j})x^{t+j} + ... + \alpha_j(b_{m-1}1_{-j})x^{m-1+j}) = 0$$

and it follows that

$$a_q \in l_R(R\langle x;\alpha\rangle(\alpha_j(b_t1_{-j})x^{t+j} + ... + \alpha_j(b_{m-1}1_{-j})x^{m-1+j}) + \\ +R\langle x;\alpha\rangle(b_mx^m)),$$

for every $j \in \mathbb{Z}$. Hence,

$$(a_px^p + ... + a_{q-1}x^{q-1})R\langle x;\alpha\rangle g(x) = (0),$$

and we obtain that $a_{q-1}R\alpha_j(1_{-j}b_m) = 0$, for every $j \in \mathbb{Z}$. Consequently,

$$(a_px^p + ... + a_{q-1}x^{q-1})R\langle x;\alpha\rangle a_{q-1}R\langle x;\alpha\rangle \sum_{i=t}^{m-1} b_ix^i = \\ f(x)(R\langle x;\alpha\rangle a_{q-1}R\langle x;\alpha\rangle)g(x) = (0).$$

By choice of $g(x)$, we have that

$$a_{q-1}R\langle x;\alpha\rangle g(x) = (0)$$

and we get

$$a_{q-1} \in l_R(R\langle x;\alpha\rangle(\alpha_j(b_t1_{-j})x^{t+j} + ... + \alpha_j(b_{m-1}1_{-j})x^{m-1+j}) + \\ +R\langle x;\alpha\rangle(b_mx^m))$$

for every $j \in \mathbb{Z}$. Now, repeating this process we obtain that

$$a_s \in l_R(R\langle x;\alpha\rangle(\alpha_j(b_t1_{-j})x^{t+j} + ... + \alpha_j(b_{m-1}1_{-j})x^{m-1+j}) + \\ +R\langle x;\alpha\rangle(b_mx^m))$$

for every $p \leq s \leq q$ and $j \in \mathbb{Z}$. Since $a_sR\langle x;\alpha\rangle b_mx^m = (0)$, for every $p \leq s \leq q$, then $(a_px^p + ... + a_qx^q)R\langle x;\alpha\rangle b_m = (0)$. This is a contradiction. $\square$

3. On partial skew Armendariz ring

Rege and Chhawchharia introduced the notion of an Armendariz ring, see [9]. A ring R is called Armendariz if whenever polynomials $\sum_{i=0}^{n} a_i x^i$, $\sum_{i=0}^{m} b_i x^i \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_i b_j = 0$, for every $0 \le i \le n$ and $0 \le j \le m$. The name Armendariz ring was chosen because Armendariz, showed that a reduced ring (i.e., a ring without nonzero nilpotent elements) satisfies this condition, see [2]. Some properties of Armendariz rings have been studied by many authors, see [2], [1], [30] and the literature quoted therein. The authors in [27], introduced the notion of skew Armendariz rings, they gave examples and investigated the properties of these rings.

In this section, we assume that R is an associative ring and α is a partial action of an infinite cyclic group G on R such that (R, α) does not necessarily have an enveloping action, unless otherwise stated.

Let S be a ring with an automorphism τ. Following [27], S is said to be τ-rigid if $a\tau(a) = 0$ implies that $a = 0$. Suppose that a partial action α of an infinite cyclic group G on R has an enveloping action (T, σ), where σ is an automorphism of T. In this case, a natural generalization of the concept mentioned above it would be $a\alpha_1(a1_{-1}) = 0 \Rightarrow a = 0$ and this is equivalent to $a\sigma(a) = 0 \Rightarrow a = 0$. The next result shows that this natural generalization implies that the partial action is a global action.

Proposition 1. *Let α be a partial action of an infinite cyclic group G on R and (T, σ) the enveloping action of (R, α), where σ is an automorphism of T. If for every $a \in R$, $a\sigma(a) = 0 \Rightarrow a = 0$, then $R = T$.*

Proof. We claim that for each $t \in T$ such that $t\sigma(t) = 0$ we have that $t = 0$. In fact, we easily have that $t1_R\alpha_1(t1_R1_{-1}) = 0$, which implies that $t1_R = 0$. Note that for each $i > 0$ we have that

$$(t\sigma^i(1_R))\sigma(t\sigma^i(1_R)\sigma^{i-1}(1_R)\sigma^i(1_R)) = 0.$$

Thus, we obtain that $\sigma^{-i}(t\sigma^i(1_R))\sigma(\sigma^{-i}(t\sigma^i(1_R)))1_R\sigma(1_R) = 0$, which implies that $\sigma^{-i}(t\sigma^i(1_R))\sigma(\sigma^{-i}(t\sigma^i(1_R))) = 0$ and by assumption we have that $\sigma^{-1}(t\sigma^i(1_R)) = 0$. Hence, $t\sigma^i(1_R) = 0$, for every $i \ge 0$.

Now, let $b = t\sigma^{-i}(1_R)$, for $i > 0$. Then $b\sigma(b\sigma^{-i}(1_R)\sigma^{-i-1}(1_R)) = 0$. Proceeding with similar method as above we obtain that $t\sigma^{-i}(1_R) = 0$, for every $i > 0$. So, $tT = 0$ and by Remark 2.5 of [16] we have that $t = 0$.

Let $T^1 = T \oplus Z$ with usual sum and multiplication defined by the rule $(a, z)(b, z_1) = (ab + az_1 + bz, zz_1)$. Note that $(0, 1)$ is the identity of T^1. We can extend the automorphism σ of T to T^1 by the rule: $\sigma(a, z) =$

$(\sigma(a), z)$. We claim that T^1 is a σ-rigid ring, i.e, if $(a, z) \in T^1$ such that $(a, z)\sigma(a, z) = 0$, then we have that $(a, z) = 0$. In fact, for each $(a, z) \in T^1$ such that $(a, z)\sigma(a, z) = (0, 0)$ we have that $a\sigma(a) = 0$. Thus, $a = 0$ and we obtain that $(a, z) = (0, 0)$. Hence, by ([27], Corollary 4) and ([24], Lemma 4) we have that all the idempotents of T^1 are invariant by σ. Since for any idempotent $e \in T$, the element $(e, 0)$ is an idempotent in T^1, then $\sigma(e) = e$. So, $\sigma(1_R) = 1_R$ and we have that $R = T$. □

Remark 1. Let α be a partial action of an infinite cyclic group G on R with an enveloping action (T, σ), where σ is an automorphism of T. It is natural to ask if $R[x; \alpha]$ were reduced then we would have that $R = T$. This is not true in general as show the following example: Let K be a field, $\{e_i : i \in \mathbb{Z}\}$ a set of central orthogonal idempotents, $T = \oplus_{i\in\mathbb{Z}} K e_i$ with an automorphism σ defined by $\sigma(e_i) = e_{i+1}$, for every $i \in \mathbb{Z}$ and $R = Ke_0$. We clearly have that the automorphism σ induces a partial action of the group $G =< \sigma >$ as follows: $S_0 = R$, $S_i = 0$, for every $i \in \mathbb{Z}\backslash\{0\}$ and the maps $\alpha_0 = id_R$ and $\alpha_i = 0$, for every $i \in \mathbb{Z}\backslash\{0\}$. Note that $R[x; \alpha] = R$ and $R \subsetneqq T$.

In the next proposition we give necessary and sufficient conditions for partial skew polynomial rings to be reduced rings and this generalizes ([27], Proposition 3).

Proposition 2. *$R[x; \alpha]$ is reduced if and only if for every $a_n \in S_n$ with $\alpha_{-n}(a_n)a_n = 0$, $n \geq 0$ implies that $a_n = 0$.*

Proof. Suppose that $R[x, \alpha]$ is reduced. Let $a_j \in S_j$ such that $\alpha_{-j}(a_j)a_j = 0$, for $j \geq 0$. Then $a_j x^j a_j x^j = \alpha_j(\alpha_{-j}(a_j)a_j)x^{2j} = 0$. Hence, by assumption we have that $ax^j = 0$. So, $a_j = 0$.

Conversely, let $f(x) = \sum_{i=0}^{n} a_i x^i$ be a polynomial in $R[x; \alpha]$ such that $(f(x))^2 = 0$. Then, $\alpha_n(\alpha_{-n}(a_n)a_n) = 0$ and by assumption we have that $a_n = 0$. Proceeding in a similar way we obtain that $a_0 = a_1 = ... = a_n = 0$. So, $f = 0$. □

Using the Proposition 2 we can generalize the notion of σ-rigid rings as follows.

Definition 1. *Let G be an infinite cyclic group and α a partial action of G on R. We say that R is a partial α-rigid ring if for every $a \in S_n$ with $\alpha_{-n}(a)a = 0$, $n \geq 0$, we have that $a = 0$.*

Remark 2. Note that if R is partial α-rigid, then R is reduced.

The following definition generalizes ([27], Definition).

Definition 2. *We say that R is a partial skew Armendariz ring if given $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{i=0}^{m} b_i x^i$ in $R[x;\alpha]$ such that $f(x)g(x)=0$, then $\alpha_{-i}(a_i)b_j=0$, for every $0\leq i\leq n$ and $0\leq j\leq m$.*

Proposition 3. *Suppose that (R,α) has an enveloping action (T,σ), where σ is an automorphism of T. If R is a partial α-rigid ring then R is a partial skew Armendariz ring.*

Proof. Let $f(x)=\sum_{i=0}^{n} a_i x_i$ and $g(x)=\sum_{j=0}^{n} b_j x_j$ be polynomials in $R[x;\alpha]$ such that $f(x)g(x)=0$. Then $a_0b_0=0$. Note that for degree 1 we have that $a_0b_1+a_1\alpha_1(b_0 1_{-1})=0$ and we obtain that $\alpha_{-1}(a_1)b_0a_1\alpha_1(b_0 1_{-1})=0$. Thus, we have that $\alpha_{-1}(a_1\alpha_1(b_0 1_{-1}))a_1\alpha_1(b_0 1_{-1}))=\alpha_{-1}(a_1)b_0a_1\alpha_1(b_0 1_{-1})=0$. Since R is partial α-rigid then $0=a_1\alpha_1(b_0 1_{-1})=\alpha_{-1}(a_1)b_0$ and consequently $a_0b_1=0$. Now, for degree 2 we have that

$$a_0b_2+a_1\alpha_1(b_1 1_{-1})+a_2\alpha_2(b_0 1_{-2})=0 \tag{*}$$

Multiplying (*) on the right side by b_1 and using the equality $a_0b_1=a_0b_0=0$ we have that $b_0a_2\alpha_2(b_0 1_{-2})=0$. Thus, $\alpha_{-2}(a_2)b_0a_2\alpha_2(b_0 1_{-2})=0$ and we obtain that $\alpha_{-2}(a_2\alpha_2(b_0 1_{-2}))a_2\alpha_2(b_0 1_{-2})=\alpha_{-2}(a_2)b_0a_2\alpha_2(b_0 1_{-2})=0$. Hence, $a_2\alpha_2(b_0 1_{-2})=\alpha_{-2}(a_2)b_0=0$. So, $b_1a_1\alpha_1(b_1 1_{-1})=0$. Proceeding with similar method as before we have that $0=a_1\alpha_1(b_1 1_{-1})=\alpha_{-1}(a_1)b_1$ and it follows that $a_0b_2=0$. Now, using a standard induction we have the desired result. □

Now, we recall some information on rings of quotients of a semiprime ring R' that we need in the sequel. For background on this subject we refer the reader to [34], Section 24; [38], Chap. 9.

An ideal H of a semiprime ring R' is essential as a two sided ideal if $r_{R'}(H)=0$. The set of all essential ideals of R' will be denoted by $\mathcal{E}=\mathcal{E}(R')$. Note that $\mathcal{E}$ is a filter of ideals which is closed under multiplication and intersection. If I is an ideal of R', then $I\oplus Ann_{R'}(I)\in\mathcal{E}$.

Denote by $Q=Q_{\mathcal{E}}$ the ring of right quotients of R' with respect to the filter $\mathcal{E}$, i.e., the Martindale ring of right quotients of R'. Recall that the elements of Q arise from right R'-homomorphisms from $H\in\mathcal{E}$ to R': for any $H\in\mathcal{E}$ and a right R'-homomorphism $f:H\to R'$ there exists $q\in Q$ such that $qh=f(h)$, for every $h\in H$, and conversely, if $q\in Q$ there exists $F\in\mathcal{E}$ such that $qF\subseteq R'$. Also, elements $q,p\in Q$ are equal if and only if they coincide on some essential ideal of R' and if $qH=0$, for some $q\in Q$ and $H\in\mathcal{E}$, then $q=0$.

Given an ideal I of R', the closure of I in R' as defined in [22] is

$$[I] = \{x \in R' \mid \text{there exists}\, H \in \mathcal{E}(R')\, \text{with}\, xH \subseteq I\} =$$

$$\{x \in R' \mid \text{there exists}\, H \in \mathcal{E}(R')\, \text{with}\, Hx \subseteq I\}.$$

Note that $[I] = r_{R'}(r_{R'}(I))$ and if $[I] = I$, then I is a closed ideal.

Let R be a partial α-rigid ring. Then R is reduced and we have that R has a Martindale ring of right quotients Q constructed as before. Following [21] the partial action α can be extended to a partial action α^* of G on Q, where the ideals S_i^* are the extension of $[S_i]$ to Q. The ideal S_i^* for every $i \in \mathbb{Z}$ is generated by a central idempotent 1_i^*. Let Q^s be the Martindale's right symmetric ring of quotients of R, i.e, according ([31], Proposition 5.14.7) we have that $Q^s = \{q \in Q : qJ \subseteq R \; and \; Jq \subseteq R \; for \; some \; J \in \mathcal{E}\}$. Now, let $(I^*)^s = \{q \in Q : qJ \cup Jq \subseteq I \; for \; some \; J \in \mathcal{E}\}$ and using similar methods of ([21], Proposition 2.2) we have that $Q(I)^s \simeq (I^*)^s$, where $Q(I)^s$ is the Martindale's right symmetric ring of quotients of I. Moreover, if I^* is generated by a central idempotent, then we have that $(I^*)^s = I^* \cap Q^s(R)$ and we denote $(I^*)^s = I^{**}$.

By ([31], Proposition 5.14.17) we have that $1_i^* \in Z(Q) = Z(Q^s)$ and $1_i^* \in (I^*)^s$, for every $i \in \mathbb{Z}$. Thus $S_i^{**} = Q^s 1_i^*$, for every $i \in \mathbb{Z}$. Now, using similar methods of ([21], Proposition 2.3 and Theorem 3.1), we can extend the partial action α to a partial action $\alpha^{**} = \{\alpha_i^{**} : S_{-i}^{**} \to S_i^{**} : i \in \mathbb{Z}\}$ of G on Q^s and $\alpha_g^{**} = \alpha_g^*|_{S_{-i}^{**}}$, for every $i \in \mathbb{Z}$. Moreover, by ([15], Theorem 4.5) we have that (Q^s, α^{**}) has an enveloping action (T'', σ''), where σ'' is an automorphism of T''. We use these facts in the next result.

Proposition 4. *R is partial α-rigid if an only if Q^s is partial α^{**}-rigid.*

Proof. Suppose that R is partial α-rigid. Let $q \in S_n^{**}$ such that $\alpha_{-n}^{**}(q)q = 0$. Since $q \in S_{-n}^{**} \subseteq S_n^*$, then there exists an essential ideal L of R such that $qL \subseteq S_n$. Thus, for every $l \in L$ we have that $\alpha_{-n}^{**}(q)ql = 0$. By ([31], Exercise 14.4) Q^s is reduced, since R is reduced. Hence, $ql\alpha_{-n}^{**}(q) = 0$, which implies that $0 = ql\alpha_{-n}^{**}(q)\alpha_{-n}^{**}(l1_n^*) = ql\alpha_{-n}^{**}(ql)$. Since $\alpha^{**} = \alpha^*|_{S_n^{**}}$, then by ([21], Theorem 3.1) we have that α_{-n}^* restricted to S_{-n} is α_{-n} and we obtain that α^{**} restricted to S_n is α_n. So, $0 = ql\alpha_{-n}^{**}(ql) = ql\alpha_{-n}(ql)$ and by assumption we have that $ql = 0$, for every $l \in L$. Consequently, $qL = 0$. Therefore, q=0.

The converse is trivial. □

In the next result we show that partial α-rigid rings are partial skew Armendariz rings and it generalizes ([27], Corollary 4).

Theorem 2. *If R is partial α-rigid, then R is partial skew Armendariz.*

Proof. By Proposition 4, Q^s is partial α^{**}-rigid and by Proposition 3, Q^s is partial skew Armendariz. Since R is a subring of Q^s and $\alpha_i^{**}|_{S_i} = \alpha_i$, we have that R is partial skew Armendariz. □

Remark 3. By Example 4.5 the converse of the Theorem 3.9 is not true.

In the next two results we give examples of partial skew Armendariz rings. The next result generalizes ([27], Proposition 10) with similar proof.

Proposition 5. *Let D be a domain and α a partial action of an infinite cyclic group G on D. Then D is partial skew Armendariz.*

One may ask if there exits a partial action of a group G on a domain S. The next example shows that such partial action exists.

Example. Assume that D is a domain, that is not a division ring, with identity element, σ is an automorphism of D such that $\sigma^i \neq id_D$, for any $i \neq 0$ and I a two-sided ideal of D. For any integer i we define $S_i = I \cap \sigma^i(I)$ and $\alpha_i : S_{-i} \to S_i$ as the restriction of σ^{-i} to S_{-i}. Then it is easy to see that $\alpha = \{\alpha_i \mid i \in \mathbb{Z}\}$ is a partial action of the infinite cyclic group $G =< \sigma >$ on I.

Let M be a (R, R)-bimodule. Then the trivial extension of R by M is the ring $T(R, M) = R \oplus M$ with usual sum and the following multiplication: $(r, m)(s, n) = (rs, rn + ms)$. This ring is isomorphic to the ring of all matrices $\begin{bmatrix} r & m \\ 0 & r \end{bmatrix}$, where $r \in R$ and $m \in M$ with usual matrix sum and multiplication.

Let $\alpha = \{\alpha_i : S_{-i} \to S_i\}$ be a partial action of an infinite cyclic group G on R and $T(R, R)$ the trivial extension of R. Then we can extend the partial action α to $T(R, R)$ as follows: $\bar{S}_i = T(S_i, S_i)$ and $\bar{\alpha}_i : \bar{S_{-i}} \to \bar{S}_i$ with $\bar{\alpha}_i(a, b) = (\alpha_i(a), \alpha_i(b))$, for every $i \in \mathbb{Z}$. Since $T(R, 0)$ is isomorphic to R, we can identify the restriction of $\bar{\alpha}_i$ to $T(R, 0)$ with α_i, for every $i \in \mathbb{Z}$. The next proposition provides examples of partial skew Armendariz rings that are not semi-prime rings and it generalizes ([27], Proposition 15).

Proposition 6. *If R is a partial α-rigid ring, then $T(R, R)$ is a partial skew Armendariz ring.*

Proof. Let $f(x) = \sum_{i=0}^n (a_i, b_i)x^i$ and $g(x) = \sum_{j=0}^n (c_j, d_j)x^j$ be elements in $T(R, R)[x; \alpha]$ such that $f(x)g(x) = 0$. We easily have that $f(x) = (p_0, p_1)$, $g(x) = (q_0, q_1)$ and $0 = f(x)g(x) = (p_0q_0, p_0q_1 + p_1q_0)$, where $p_0 = \sum_{i=0}^n a_i x^i$, $p_1 = \sum_{i=0}^n b_i x^i$, $q_0 = \sum_{j=0}^n c_j x^j$ and $q_1 = \sum_{j=0}^n d_j x^j$.

Note that $q_0p_0 = 0$ and using similar ideas of ([27] Proposition 15) we obtain that $p_0q_1 = p_1q_0 = 0$. Hence, by Theorem 2, $\alpha_{-i}(a_i)c_j = \alpha_{-i}(a_i)d_j = \alpha_{-i}(b_i)c_j = 0$, for every $0 \leq i \leq n$ and $0 \leq j \leq n$. So, $(\alpha_{-i}(a_i), \alpha_{-i}(b_i))(c_j, d_j) = (0, 0)$, for every $0 \leq i \leq n$ and $0 \leq j \leq n$. □

A right annihilator of a non-empty subset X of a not necessarily associative ring S' is defined by $r_{S'}(X) = \{a \in S : Xa = 0\}$. Since S' is not necessarily associative, then $r_{S'}(X)$ is not necessarily a right ideal. If we have an associative ring S_1 contained in S, then for every non-empty subset Y of S_1 we have $r_{S_1}(Y) = r_S(Y) \cap S_1$. We put $rAnn_R(2^R) = \{r_R(U) : U \subseteq R\}$ and for a not necessarily associative ring S' we put analogously $rAnn_{S'}(2^{S'}) = \{r_{S'}(U) : U \subseteq S'\}$.

Lemma 2. *Let U be a non-empty subset of R. Then*

$$r_{R[x;\alpha]}(U) = r_R(U)R[x;\alpha].$$

Proof. Let $f(x) = \sum_{i=0}^n a_ix^i \in R[x;\alpha]$ such that $Uf(x) = 0$. Then $Ua_i = 0$ for every $0 \leq i \leq n$ and it follows that $a_i \in r_R(U)$ for every $0 \leq i \leq n$. Thus, $f(x) = \sum_{i=0}^n a_ix^i \in r_R(U)R[x;\alpha]$. Hence, $r_{R[x;\alpha]}(U) \subseteq r_R(U)R[x;\alpha]$ and we easily have that $r_R(U)R[x;\alpha] \subseteq r_{R[x;\alpha]}(U)$. So, $r_{R[x;\alpha]}(U) = r_R(U)R[x;\alpha]$. □

From Lemma 2 we have the maps

$$\phi : rAnn_R(2^R) \to rAnn_{R[x;\alpha]}(2^{R[x;\alpha]})$$

defined by $\phi(I) = IR[x;\alpha]$, for every $I \in rAnn_R(2^R)$ and

$$\Psi : rAnn_{R[x;\alpha]}(2^{R[x;\alpha]}) \to rAnn_R(2^R)$$

defined by $\Psi(J) = J \cap R$, for every $J \in rAnn_{R[x;\alpha]}(2^{R[x;\alpha]})$. Obviously, ϕ is injective and Ψ is surjective. Clearly ϕ is surjective if and only if Ψ is injective, and in this case ϕ and Ψ are the inverses of each other.

The following result generalizes ([11], Proposition 3.2).

Lemma 3. *The following conditions are equivalent:*
(i) R is a partial skew Armendariz ring.
(ii) $\phi : rAnn_R(2^R) \to rAnn_{R[x;\alpha]}(2^{R[x;\alpha]})$ is bijective.

Proof. Suppose that R is a partial skew Armendariz ring. It is only necessary to show that ϕ is surjective. For every $f(x) = \sum_{i=0}^n a_ix^i \in R[x;\alpha]$ we define $C_{f(x)} = \{\alpha_{-i}(a_i),\ 0 \leq i \leq n\}$ and for a subset S of $R[x;\alpha]$ we denote the set $\bigcup\limits_{f(x)\in S} C_{f(x)}$ by C_S. We claim that $r_{R[x;\alpha]}(f(x)) =$

$r_{R[x;\alpha]}(C_{f(x)})$. In fact, let $g(x) = \sum_{i=0}^{m} b_i x^i \in r_{R[x;\alpha]}(f(x))$. Then, we have that $f(x)g(x) = 0$. By assumption, we have that $\alpha_{-i}(a_i)b_j = 0$, for every $0 \leq i \leq n$ and $0 \leq j \leq m$. Thus, $g(x) \in r_{R[x;\alpha]}(C_{f(x)})$.

On the other hand, let $h(x) = \sum_{i=0}^{p} c_i x^i$ be an element in $R[x;\alpha]$ such that $C_{f(x)}h(x) = 0$. Then we have that $\alpha_{-i}(a_i)c_k = 0$, for every $0 \leq i \leq n$ and $0 \leq k \leq p$, which implies that $f(x)h(x) = 0$.

Since R is partial skew Armendariz, then $r_{R[x;\alpha]}(S) = r_{R[x;\alpha]}(\underset{f(x)\in S}{\cup} C_{f(x)})$ and by Lemma 2 we have that $r_{R[x;\alpha]}(C_{f(x)}) = r_R(C_{f(x)})R[x;\alpha]$. Hence,

$$r_{R[x;\alpha]}(S) = \underset{f(x)\in S}{\cap} r_{R[x;\alpha]}(f(x)) = \underset{f(x)\in S}{\cap} r_{R[x;\alpha]}(C_{f(x)}) =$$

$$= (\underset{f(x)\in S}{\cap} r_R(C_{f(x)}))R[x;\alpha] = r_R(C_S)R[x;\alpha].$$

So, ϕ is surjective.

Conversely, let $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{i=0}^{m} b_i x^i$ be elements in $R[x;\alpha]$ such that $f(x)g(x) = 0$. Then, by assumption, $g(x) \in r_{R[x,\alpha]}(f(x)) = BR[x;\alpha]$, for some right ideal B of R. Thus, we have that, $b_i \in B \subset r_{R[x;\alpha]}(f(x))$, for every $0 \leq i \leq m$. Hence, $\alpha_{-i}(a_i)b_j = 0$, for every $0 \leq i \leq n$ and $0 \leq j \leq m$. So, R is a partial skew Armendariz ring. □

The next lemma generalizes ([27], Corollary 19) and the proof is similar.

Lemma 4. *Let R be a partial skew Armendariz ring and $e = \sum_{i=0}^{n} e_i x^i \in R[x;\alpha]$. If $e^2 = e$, then $e = e_0$.*

The following definition appears in [26].

Definition 3. *A ring R is called a Baer ring, if the left annihilator of each subset of R is generated by an idempotent.*

Remark 4. Note that the definition of a Baer ring is left-right symmetric.

The notion of right ideals in non-associative rings is the same as in associative rings. Moreover, if S' is not necessarily associative, we have for each $a \in S'$ the set $aS' : \{as : s \in S'\}$. We use these facts in the next results.

The proofs of the next four theorems are similar with ([11], Theorem 3.6), ([11], Theorem 3.8), ([11], Proposition 3.9) and ([12], Theorem 2.8) respectively, and we put their proofs here for the reader's convenience.

The next theorem generalizes ([11], Theorem 3.6).

Theorem 3. *Suppose that R is a partial skew Armendariz ring. Then the following conditions are equivalent:*

(i) R is a Baer ring.

(iii) $R[x;\alpha]$ satisfies the following property: for every non-empty subset U of $R[x;\alpha]$ we have that $r_{R[x;\alpha]}(U) = eR[x;\alpha]$, for some idempotent $e \in R$.

Proof. Suppose that R is a Baer ring. Let $\varnothing \neq U$ be a subset of $R[x;\alpha]$ and $f(x) = \sum_{i=0}^{n} a_i x^i \in U$. We define $C_{f(x)} = \{\alpha_{-i}(a_i),\ for\ every\ 0 \leq i \leq n\}$, and $C_U = \bigcup_{f(x)\in U} C_{f(x)}$. By assumption, we have that $r_R(C_U) = eR$, with $e^2 = e$. It is easy to see that $eR[x;\alpha] \subset r_{R[x;\alpha]}(U)$. By Lemma 3, we have

$$r_{R[x;\alpha]}(U) = r_R(C_U)R[x;\alpha] = eR[x;\alpha].$$

Conversely, let $\varnothing \neq A$ be a subset of R. Then, by assumption we have that $r_{R[x;\alpha]}(A) = eR[x;\alpha]$, where $e^2 = e$. By Lemma 4, $e = e_0$ is a constant polynomial. Thus,

$$r_{R[x;\alpha]}(A) = eR[x;\alpha] = e_0R[x;\alpha].$$

Hence, $r_{R[x;\alpha]}(A) \cap R = e_0R[x;\alpha] \cap R = e_0R$. So, R is a Baer ring. □

Definition 4. *A ring R is called left (right) p.p if the left (right) annihilator of every element is generated by an idempotent as a left (right) ideal.*

The next theorem generalizes ([11], Theorem 3.8).

Theorem 4. *Suppose that R is a partial skew Armendariz ring. Then the following conditions are equivalent:*

(i) R is a right p.p-ring.

(ii) $R[x;\alpha]$ satisfies the following property: for every polynomial $f(x)$ of $R[x;\alpha]$ we have $r_{R[x;\alpha]}(f(x)) = eR[x;\alpha]$, for some idempotent $e \in R$.

We recall that a ring S satisfies the ascending chain condition on right annihilator ideals if for any chain $r_S(Y_1) \subseteq r_S(Y_2) \subseteq ...$, there exists $i \geq 1$ such that $r_S(Y_i) = r_S(Y_{i+p})$, for all $p \geq 0$, where Y_j are non-empty subsets of S for every $j \geq 0$. Moreover, for a not necessarily associative rings S' satisfies the ascending chain conditions on right annihilators if $r_{S'}(Y'_1) \subseteq r_{S'}(Y'_2) \subseteq ...$, there exists $i \geq 1$ such that $r_{S'}(Y'_i) = r_{S'}(Y'_{i+p})$, for all $p \geq 0$, where Y'_j are non-empty subsets of S' for every $j \geq 0$. In the next proposition, we suppose that R is a partial skew Armendariz ring to study the transfer of ascending chain condition on right annihilators property between R and $R[x;\alpha]$ and this generalizes ([11], Proposition 3.9).

Theorem 5. *Suppose that R is a partial skew Armendariz ring. Then the following conditions are equivalent:*

(*i*) *R satisfies ascending chain condition on right annihilator ideals.*

(*ii*) *$R[x;\alpha]$ satisfies the ascending chain condition on right annihilators.*

Proof. Suppose that $R[x;\alpha]$ satisfies the ascending chain condition on right annihilators. We consider the chain, $r_R(U_1) \subset r_R(U_2) \subset ..$, where $U_i \subset R$, for every $i \geq 1$. We claim that $r_{R[x;\alpha]}(U_i) \subset r_{R[x;\alpha]}(U_{i+1})$. In fact, let $g(x) = \sum_{i=0}^{n} c_i x^i$ be an element of $R[x;\alpha]$, such that $U_i g(x) = (0)$. Then, $U_i c_j = (0)$, and we obtain that $U_{i+1} c_j = (0)$. Hence, by assumption there exists $l \geq 1$ such that $r_{R[x;\alpha]}(U_l) = r_{R[x;\alpha]}(U_{l+p})$, for every $p \in \mathbb{N}$. So,

$$r_R(U_l) = r_{R[x;\alpha]}(U_l) \cap R = r_{R[x;\alpha]}(U_{l+p}) \cap R = r_R(U_{l+p}),$$

for every $p \in \mathbb{N}$.

Conversely, suppose that $r_{R[x;\alpha]}(V_1) \subset r_{R[x;\alpha]}(V_2) \subset ....$ By Lemma 3, $r_{R[x;\alpha]}(V_i) = r_R(C_{V_i})R[x;\alpha]$, where $C_{V_i} = \underset{f(x)\in V_i}{\cup} C_{f(x)}$,

$$C_{f(x)} = \{\alpha_{-i}(a_i),\ for\ every\ 0 \leq i \leq m\}$$

and $f(x) = \sum_{i=0}^{m} a_i x^i$. We claim that $r_R(C_{V_i}) \subset r_R(C_{V_{i+1}})$. In fact, let $y \in r_R(C_{V_i})$. Then we have that $C_{V_i} y = 0$. Thus, $f(x)y = 0$, for every $f(x) \in V_i$. Hence, $C_{V_{i+1}} y = 0$ and by assumption, there exists $n \geq 1$ such that $r_R(C_{V_n}) = r_R(C_{V_{n+k}})$, for every $k \in \mathbb{N}$. So, $r_{R[x;\alpha]}(V_n) = r_{R[x;\alpha]}(V_{n+k})$, for every $k \in \mathbb{N}$. $\square$

In [18], Faith called a ring R right zip if the right annihilator $r_R(U)$ of a non-empty subset U of R is zero implies that $r_R(Y) = 0$ for every non-empty finite subset $Y \subseteq U$. Equivalently, for a left ideal L of R with $r_R(L) = 0$, there exists a finitely generated left ideal $L_1 \subseteq L$ such that $r_R(L_1) = 0$. R is zip if it is right and left zip. The concept of zip rings was initiated by Zelmanowitz [39] and appeared in various papers [3], [6], [7], [8], [18] [19], and references therein. Zelmanowitz stated that any ring satisfying the descending chain condition on right annihilators ideals is a right zip ring (although not so-called at that time), but the converse does not hold. Extensions of zip rings were studied by several authors. Beachy and Blair [3] showed that if R is a commutative zip ring, then the polynomial ring $R[x]$ over R is zip. The authors in [25] proved the following result: suppose that R is Armendariz ring. Then R is a right (left) zip ring if and only if $R[x]$ is a right (left) zip ring. The next result generalizes ([12], Theorem 2.8).

Theorem 6. *Suppose that R is a partial skew Armendariz ring. Then the following conditions are equivalent:*

(i) R is a right zip ring.

(ii) $R[x;\alpha]$ satisfies the following property: for every non-empty subset U of $R[x;\alpha]$ such that $r_{R[x;\alpha]}(U)=0$, we have that $r_{R[x;\alpha]}(Y)=0$ for any non-empty finite subset Y of X.

Proof. Suppose that R is a right zip ring. Let U be a non-empty subset of $R[x;\alpha]$ such that $r_{R[x;\alpha]}(U)=0$. Then, by Lemma 3, $r_{R[x;\alpha]}(U)=r_R(C_U)R[x;\alpha]$, where $C_U=\cup_{f(x)\in U}C_{f(x)}$ and $C_{f(x)}=\{\alpha_{-i}(a_i):0\leq i\leq n\}$ with $f(x)=\sum_{i=0}^n a_ix^i\in U$. Thus, $r_R(C_U)=0$ and by assumption for any non-empty finite subset $W=\{\alpha_{-i_1}(a_{i_1}),...,\alpha_{-i_n}(a_{i_n})\}$ of C_U we have that $r_R(W)=0$. For every $\alpha_{-i_j}(a_{i_j})\in W$ there exists $g_{a_{i_j}}(x)\in U$ such that some of the coefficients of $g_{a_{i_j}}(x)$ are a_{i_j}, for every $1\leq j\leq n$. Let U_0 be a minimal subset of U such that $g_{a_{i_j}}(x)\in U_0$, for every $1\leq j\leq n$. Then U_0 is non-empty finite subset of U. We denote $W_0=\cup_{f(x)\in U_0}(C_{f(x)})$ and note that $W\subseteq W_0$. Hence, $r_R(W_0)\subseteq r_R(W)=0$ and, by Lemma 3, $r_{R[x;\alpha]}(U_0)=r_R(W_0)R[x;\alpha]$. So, $r_{R[x;\alpha]}(U_0)=0$.

Conversely, let Y be a non-empty subset of R such that $r_R(Y)=0$ and $f(x)=\sum_{i=0}^n a_ix^i\in r_{R[x;\alpha]}(Y)$. Then we have that $a_i\in r_R(Y)=0$ and it follows that $f(x)=0$. Thus, by assumption for any non-empty finite subset $Y_1=\{y_0,...,y_n\}$ of Y we have that $r_{R[x;\alpha]}(Y_1)=0$. Hence, $r_R(Y_1)=r_{R[x;\alpha]}(Y_1)\cap R=(0)$. □

4. Examples

In this section, we give an example to show that the partial skew Laurent polynomial rings and the partial skew polynomial rings are not necessarily associative rings if the base ring satisfy either Baer or quasi-Baer or p.q. Baer or p.p. Moreover, for a ring R with a partial action α of a group G with enveloping action (T,β) we study the transfer of Baer property, quasi-Baer property, p.q. Baer property and p.p property between R and T.

We begin with the following proposition.

Proposition 7. *Let α be a partial action of an infinite cyclic group G on R and (T,σ) its enveloping action, where σ is an automorphism of T. If T is σ-rigid, then R is partial α-rigid.*

Proof. Let $a \in S_n$ such that $\alpha_{-n}(a)a = 0$. Then $\sigma^{-n}(a)a = 0$, which implies that $a\sigma^n(a) = 0$. Thus, by ([28], Lemma 4) we have that $a^2 = 0$. So, $a = 0$, since T is reduced. □

The next example shows that the converse of the proposition above is not true.

Example 1. Let K be a field and $T = \oplus_{i\in\mathbb{Z}} Ke_i$, where $\{e_i, i \in \mathbb{Z}\}$ are orthogonal central idempotents. We define an action of an infinite cyclic group G generated by σ as follows: $\sigma|_K = id_K$ and $\sigma(e_i) = e_{i+1}$. Assume that $R = Ke_0$ and we have a partial action α of G on R. Note that R is partial α-rigid, but T is not σ-rigid, because $e_2\sigma(e_2) = e_2e_3 = 0$ and $e_2 \neq 0$.

Let T' be a ring and σ' an endomorphism of T'. The skew polynomial ring of endomorphism type $T'[x;\sigma']$ is the set of all finite formal sums $\sum_{i=0}^{n} a_i x^i$ with usual sum and the multiplication rule is $ax^ibx^j = a\sigma^i(b)x^{i+j}$. Next, we recall the definition that appears in ([27], Definition).

Definition 5. *Let T' be a ring and σ' an endomorphism of T'. The ring T' is said to be skew Armendariz ring if given $f(x) = \sum_{i=0}^{n} a_i x^i$ and $g(x) = \sum_{i=0}^{m} b_i x^i$ in $T[x;\sigma']$ such that $f(x)g(x) = 0$, then $a_i\sigma^i(b_j) = 0$, for every $0 \le i \le n$ and $0 \le j \le m$.*

When a partial action α of G on R has an enveloping action (T, σ) one may ask if R were a partial skew Armendariz ring, then T would be a skew Armendariz ring, but the next example shows that this is not true in general.

Example 2. Let K, T, σ, R and α as in the Example 1. We clearly have that R is a partial skew Armendariz ring, but T is not skew Armendariz since for $f = e_{-1} + e_{-1}x$ and $g = -e_{-2} + e_{-1}x$ in $T[x;\sigma]$ we obtain that $fg = 0$ and $e_{-1}e_{-1} \neq 0$.

The next example shows that the partial skew Armendariz property does not imply the associativity of the partial skew polynomial rings and partial skew Laurent polynomial rings. Moreover, the next example shows that the converse of Theorem 3.9 is not true.

Example 3. Let $R = K[X, Y]/(X^2, Y^2)$, where K is a field. Then $R = K1 + Kx + Ky + Kxy$, where x and y represent the classes of X and Y in R. Note that R is a four dimensional K-vector space. Let $G =< \sigma >$ be an infinite cyclic group generated by σ and we define the partial action α of G on R as follows: let $I = Kx + Kxy$, $S_i = I$ and $\alpha_i : S_{-i} \to S_i$

defined by $\alpha_i(x) = xy$, $\alpha_i(xy) = x$ and $\alpha_i|_K = id_K$, for every $i \in \mathbb{Z} \setminus \{0\}$ (by definition $S_0 = R$ and $\alpha_0 = id_R$). We claim that R is a partial skew Armendariz ring. In fact, let $f(z) = (\gamma_0+\gamma_1x+\gamma_2y+\gamma_3xy)+(a_0x+b_0xy)z$ and $g(z) = (\theta_0+\theta_1x+\theta_2y+\theta_3xy)+(a_1x+b_1xy)z$ be polynomials of degree 1 in $R[z;\alpha]$ such that $f(z)g(z) = 0$. Then $(\gamma_0 + \gamma_1x + \gamma_2y + \gamma_3xy)(\theta_0 + \theta_1x + \theta_2y + \theta_3xy) = 0$, which implies $\gamma_0\theta_0 = 0$. Thus, we have the followin g cases:

Case 1: If $\gamma_0 = 0$ and $\theta_0 \neq 0$, then we have that $f(z) = 0$.

Case 2: If $\gamma_0 \neq 0$ and $\theta_0 = 0$, then we have that $g(z) = 0$.

Case 3: If $\gamma_0 = \theta_0 = 0$, then we have that $(\gamma_1\theta_2 + \gamma_2\theta_1)xy = 0$ and $\gamma_2a_1x = b_0\theta_2xy = 0$. Thus, $\gamma_2a_1 + b_0\theta_2 = 0$, $\gamma_2a_1 = 0$ and $b_0\theta_2 = 0$, which implies either $\gamma_2 = 0$ or $a_1 = 0$ and either $b_0 = 0$ or $\theta_2 = 0$. Hence, we have the following cases:

Case 3.1: If $\gamma_2 = 0$, $a_1 \neq 0$, $b_0 = 0$ and $\theta_2 \neq 0$, then we have that $[(\gamma_1x + \gamma_3xy) + (a_0x)z][(\theta_1x + \theta_2y + \theta_3xy) + (a_1x + b_1xy)z] = 0$. Thus, $(\gamma_1x + \gamma_3xy)(\theta_1x + \theta_2y + \theta_3xy) = 0$, $\alpha_1(\alpha_{-1}(a_0x)(\theta_1x + \theta_2y + \theta_3xy)) = \alpha_1((a_0xy)(\theta_1x + \theta_2y + \theta_3xy)) = 0$ and $\alpha_1(\alpha_{-1}(a_0x)(a_1x + b_1xy)) = 0$.

Case 3.2: If $\gamma_2 = 0$, $a_1 \neq 0$, $b_0 \neq 0$, $\theta_2 = 0$, then we have that $[(\gamma_1x + \gamma_3xy) + (a_0x + b_0xy)z][(\theta_1x + \theta_3xy) + (a_1x + b_1xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}((a_0x + b_0xy))(\theta_1x + \theta_3xy)) = \alpha_1((a_0xy + b_0x)(\theta_1x + \theta_3xy)) = 0$, $(\gamma_1x + \gamma_3xy)(a_1x + b_1xy) = 0$, $(\gamma_1x + \gamma_3xy)(\theta_1x + \theta_3xy) = 0$ and $\alpha_1(\alpha_{-1}((a_0x + b_0xy))(a_1x + b_1xy)) = 0$.

Case 3.3: If $\gamma_2 \neq 0$, $a_1 = 0$, $\theta_2 = 0$, $b_0 \neq 0$, then we have that $[(\gamma_1x + \gamma_2y + \gamma_3xy) + (a_0x)z][(\theta_1x + \theta_2y + \theta_3xy) + (b_1xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0x)(\theta_1x + \theta_2y + \theta_3xy)) = \alpha_1(a_0xy(\theta_1x + \theta_2y + \theta_3xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0x)(b_1xy)) = 0$, $(\gamma_1x + \gamma_2y + \gamma_3xy)(b_1xy) = 0$ and $(\gamma_1x + \gamma_2y + \gamma_3xy)(\theta_1x + \theta_2y + \theta_3xy) = 0$.

Case 3.4: If $\gamma_2 \neq 0$, $a_1 = 0$, $\theta_2 \neq 0$, $b_0 = 0$, then we have that $[(\gamma_1x + \gamma_2y + \gamma_3xy) + a_0xz][(\theta_1x + \theta_2y + \theta_3xy) + (b_1xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0x)(\theta_1x + \theta_3xy)) = \alpha_1((a_0xy)(\theta_1x + \theta_3xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0x)(b_1xy)) = 0$, $(\gamma_1x + \gamma_2y + \gamma_3xy)(b_1xy) = 0$ and $(\gamma_1x + \gamma_2y + \gamma_3xy)(\theta_1x + \theta_3xy) = 0$.

Case 3.5: If $\gamma_2 = 0$, $a_1 = 0$, $\theta_2 \neq 0$, $b_0 = 0$, then we have that $[(\gamma_1x + \gamma_3xy) + (a_0x)z][(\theta_1x + \theta_2 + \theta_3xy) + (b_1xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0x)(\theta_1x + \theta_2y + \theta_3xy)) = \alpha_1((a_0xy)(\theta_1x + \theta_2y + \theta_3xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0x)(b_1xy)) = 0$, $(\gamma_1x + \gamma_3xy)(b_1xy) = 0$ and $(\gamma_1x + \gamma_3xy)(\theta_1x + \theta_3xy) = 0$.

Case 3.6: If $\gamma_2 = 0$, $a_1 = 0$, $\theta_2 = 0$, $b_0 \neq 0$, then we have that $[(\gamma_1x + \gamma_3xy) + (a_0x + b_0xy)z][(\theta_1x + \theta_3xy) + (b_1xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0x + b_0xy)(\theta_1x + \theta_3xy)) = \alpha_1((a_0xy + b_0x)(\theta_1x + \theta_3xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0x + b_0xy)(b_1xy)) = 0$, $(\gamma_1x + \gamma_3xy)(b_1xy) = 0$ and $(\gamma_1x +$

$\gamma_3 xy)(\theta_1 x + \theta_3 xy) = 0$.

Case 3.7: If $\gamma_2 = 0$, $a_1 \neq 0$, $\theta_2 = 0$, $b_0 = 0$, then we have that $[(\gamma_1 x + \gamma_3 xy) + (a_0 x)z][(\theta_1 x + \theta_3 xy) + (a_1 x + b_1 xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0 x)(\theta_1 x + \theta_3 xy)) = \alpha_1((a_0 xy)(\theta_1 x + \theta_3 xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0 x)(a_1 x + b_1 xy)) = 0$, $(\gamma_1 x + \gamma_3 xy)(a_1 x + b_1 xy) = 0$ and $(\gamma_1 x + \gamma_3 xy)(\theta_1 x + \theta_3 xy) = 0$.

Case 3.8: If $\gamma_2 \neq 0$, $a_1 = 0$, $\theta_2 = 0$, $b_0 = 0$, then we have that $[(\gamma_1 x + \gamma_2 y + \gamma_3 xy) + (a_0 x)z][(\theta_1 x + \theta_3 xy) + (b_1 xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0 x)(\theta_1 x + \theta_3 xy)) = \alpha_1((a_0 xy)(\theta_1 x + \theta_3 xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0 x)(b_1 xy)) = 0$, $(\gamma_1 x + \gamma_2 y + \gamma_3 xy)(b_1 xy) = 0$ and $(\gamma_1 x + \gamma_2 y + \gamma_3 xy)(\theta_1 x + \theta_3 xy) = 0$.

Case 3.9: If $\gamma_2 = 0$, $a_1 = 0$, $\theta_2 = 0$, $b_0 = 0$, then we have that $[(\gamma_1 x + \gamma_3 xy) + (a_0 x)z][(\theta_1 x + \theta_3 xy) + (b_1 xy)z] = 0$. Thus, $\alpha_1(\alpha_{-1}(a_0 x)(\theta_1 x + \theta_3 xy)) = \alpha_1((a_0 xy)(\theta_1 x + \theta_3 xy)) = 0$, $\alpha_1(\alpha_{-1}(a_0 x)(b_1 xy)) = 0, (\gamma_1 x + \gamma_3 xy)(b_1 xy) = 0$ and $(\gamma_1 x + \gamma_3 xy)(\theta_1 x + \theta_3 xy) = 0$.

So, the result follows for polynomials of degree 1.

Next, let $f(z) = \sum_{i=0}^{n} a_i z^i$ and $g(z) = \sum_{i=0}^{n} b_i z^i$ be polynomials of degree n in $R[z;\alpha]$ such that $f(z)g(z) = 0$. Then $a_0 b_0 = 0$. Since $I^2 = 0$, we have that $\alpha_i(\alpha_{-i}(a_i)b_j) = 0$, for every $1 \leq i \leq n$ and $1 \leq j \leq n$. Thus, we have that $0 = f(z)g(z) = a_0 b_0 + (a_0 + \alpha_1(\alpha_{-1}(a_1)b_0))z + (a_0 b_2 + \alpha_2(\alpha_{-2}(a_2)b_0)z^2 + \dots + (a_0 b_n + \alpha_n(\alpha_{-n}(a_n)b_0))z^n$. Now, we can apply the methods used to prove the result for polynomials of degree 1 to the degree 0 and 1, degree 0 and 2, and so on. So, R is partial skew Armendariz. Moreover, the ring $R[z;\alpha]$ is not associative, because $((y)(xyz))y = 0$ and $(y)((xyz)y) = (y)(xz) = xyz \neq 0$.

The following definition appears in [26] and [5].

Definition 6. *(i) A ring R is called a left (right) quasi-Baer ring, if the left (right) annihilator of any left (right) ideal is generated by an idempotent as a left (right) ideal.*

(ii) A ring R is called a left (right) p.q.-Baer ring if the left (right) annihilator of any principal left (right) ideal is generated by an idempotent as a left (right) ideal.

Remark 5. It is well-known that Baer rings $\Rightarrow$ left (right) quasi-Baer rings $\Rightarrow$ left (right) p.q-Baer rings and that Baer rings $\Rightarrow$ left (right) p.p-rings. In the articles [5], [27], [28] and the literature quoted therein we can find examples where the converse of each arrow is not true in general.

In [23] ([35]), the author defined u.p. semigroups (groups) i.e, a semigroup (group) G' is an u.p semigroup (group) if for any non-empty subsets A and B of G', there exists at least one $y \in G'$ that has an unique representation of the form $y = ab$ with $a \in A$ and $b \in B$. It is not difficult to show that any cyclic group is an u.p group. The author in [23] used u.p. groups to study groups rings that are Baer and p.p rings. It is convenient to remark that this concept is used to the characterization of zero divisors in group rings and semigroups rings.

One may ask if Baer, quasi-Baer, p.q-Baer and p.p-rings would imply the associativity of the partial skew Laurent polynomial ring, but the next example shows that this is false in general and part of the example below is contained in ([32], Proposition 2.1).

We denote the Jacobson radical of a ring S_1 by $J(S_1)$.

Example 4. Let H be a cyclic group of order 5 and F a field of characteristic 5. Then, by ([23], Theorem 2) we have that that FH is a Baer ring. Since H is cyclic it follows that $J(FH)^3$ has dimension 2 over F and we have that $J(FH) \supsetneqq J(FH)^3$. Note that $\{1, (y-1), (y-2)^2, (y-1)^3, (y-1)^4\}$ is an F-basis of FH. It is easy to see that $J(FH)^3 = FH(y-1)^3$ is the subspace generated by $(y-1)^3$ and $(y-1)^4$. Let G be an infinite cyclic group generated by σ, $I = J(FH)^3$ and α the partial action of G on FH defined as follows: $S_i = S_{-i} = I$ for every $0 \neq i \in \mathbb{Z}$ and $\alpha_i : S_{-i} \to S_i$ given by $\alpha_i((y-1)^3) = (y-1)^4$ and $\alpha_i((y-1)^4) = (y-1)^3$ (by definition $S_0 = FH$ and α_0 is the identity automorphism of FH). In this case $FH *_\alpha G = FH < x; \alpha >$. We claim that $FH < x; \alpha >$ is not associative. In fact, let $a = (y-1) + (y-1)^3 x$ (note that $(y-1) \in J(FH) \subseteq FH = S_0$ and $(y-1) \notin J(FH)^3 = I$). Then we obtain that $a.a = (y-1)^2 + (y-1)^4 x$. Thus, we have that $(aa)a = (y-1)^3 + (y-1)^3 x$ while $a(aa) = (y-1)^3$. So, $(FH) < x; \alpha >$ is not associative and consequently $(FH)[x; \alpha]$ is not associative by the same reasons.

Let G be a group and α a partial action of G on R. Suppose that (R, α) has enveloping action (T, β). One may ask if R is either Baer or quasi-Baer or p.q-Baer or p.p. implies that T is either Baer or quasi-Baer or p.q-Baer or p.p, respectively. The next example shows this is not true in general.

Example 5. Consider T, R, σ and α as in the Example 1. We clearly have that R is Baer but T is not Baer, quasi-Baer , right (left) p.q.Baer and right (left) p.p. In fact, let $y = e_1$. Then $r_T(e_1) = \oplus_{i \in \mathbb{Z},\, i \neq 1} K e_i$ that is not generated by an idempotent.

The proof of the following lemma is standard an so we omit it here.

Lemma 5. *Let S be a ring and I an ideal of S generated by a central idempotent. Then the following statements hold:*

(i) If S is Baer, then I is Baer.

(ii) If S is quasi-Baer, then I is quasi-Baer.
(iii) If S is right (left) p.q.-Baer, then I is right (left) p.q.-Baer.
(iv) If S is right (left) p.p, then I is right (left) p.p.

By ([20], Proposition 1.2) a partial action α of a group G on R with enveloping action (T,β) is of finite type if there exists $g_1,...,g_n \in G$ such that $T=\sum_{i=1}^{n}\beta_{g_i}(R)$. Moreover, T has a unit 1_T.

In the next result we suppose that the partial action α of G on R with enveloping action (T,β) is of finite type.

Theorem 7. *(i) R is Baer if and only if T is Baer.*
(ii) R is quasi-Baer if and only if T is quasi-Baer.
(iii) R is right (left) p.q.-Baer if and only if T is right (left) p.q-Baer.
(iv) R is right (left) p.p if and only if T is right (left) p.p.

Proof. We only need to prove (i) because the other ones use similar ideas. Suppose that R is Baer and let $\emptyset \neq Y$ be any subset of T. Then by ([20], Proposition 1.10 and Remark 1.11) we have that $T=R\oplus A_1\oplus ...\oplus A_n$ and $1_T=1_R+f_1+f_2+...+f_n$, where $1_R, f_1,...,f_n$ are orthogonal central idempotents with $f_i\in A_i$ for every $1\leq i\leq n$. Thus, for any $y\in Y$ we have that $y=y1_R+yf_1+...+yf_n\in Y1_R\oplus Yf_1\oplus ...\oplus Yf_n$. Hence, for every $a\in r_T(Y)$ and $y\in Y$, we have that $ya=y1_Ra1_R+yf_1af_1+...+yf_naf_n=0$, which implies $y1_Ra1_R=yf_1af_1=...=yf_naf_n=0$. So, $a1_R\in r_R(Y1_R)$, $af_i\in r_{A_i}(Yf_i)$ for every $1\leq i\leq n$ and since $a=a1_R+af_1+...+af_n$, we obtain that $a\in r_R(Y1_R)\oplus r_{A_1}(Yf_1)\oplus ...\oplus r_{A_n}(Yf_n)$.

On the other hand, for every $z\in r_R(Y1_R)\oplus r_{A_1}(Yf_1)\oplus ...\oplus r_{A_n}(Yf_n)$ we have that $yz=0$ for every $y\in Y$. Thus, $r_T(Y)=r_R(Y1_R)\oplus r_{A_1}(Yf_1)\oplus ...\oplus r_{A_n}(Yf_n)$. By the fact that the Baer property is preserved by isomorphisms and using Lemma 5, we obtain that $r_R(Y1_R)=eR$, $r_{A_i}(Yf_i)=e_iA_i$, for $1\leq i\leq n$. Hence, $r_T(Y)=eR\oplus e_1A_1\oplus ...\oplus e_nA_n$. So, $r_T(Y)=(e+e_1+...+e_n)T$ and we easily have that $e+e_1+...+e_n$ is an idempotent since $ee_i=e_je_k=0$, for every $1\leq i,j,k\leq n$ with $j\neq k$.

By Lemma 5 the converse follows. □

One may ask if a partial action α of an infinite cyclic group G on R with enveloping action (T,σ) such that α is of finite type we would have: R is partial α-rigid $\Leftrightarrow$ T is σ-rigid and R is partial skew Armendariz $\Leftrightarrow$ T is skew Armendariz. The next example shows that these facts are not true in general.

Example 6. Assume that K is a field and $T=ke_1\oplus Ke_2\oplus Ke_3$, where $\{e_1,e_2,e_3\}$ are orthogonal central idempotents. We define an action of an infinite cyclic group G generated by σ as follows: $\sigma|_k=id_K$ and

$\sigma(e_i) = e_{i+1}$. Let $R = Ke_1$. We easily have a partial action α of G on R. Note that (T, σ) is the enveloping action of (R, α), α is of finite type, R is partial α-rigid and partial skew Armendariz, but T is not σ-rigid because $e_2\sigma(e_2) = 0$ and $e_2 \neq 0$. We claim that T is not skew Armendariz. In fact, let $f(x) = e_1 + e_1x$ and $g(x) = -e_3 + e_1x$ be polynomials in $T[x; \sigma]$. Then, we have that $f(x)g(x) = 0$ and $e_1e_1 \neq 0$. So, T is not a skew Armendariz ring.

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Received by the editors: 09.02.2011
and in final form 09.02.2011.