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Domination on the vertices of labeled graphs

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Abstract. In this paper we introduce and study a domination relation on vertices of vertex-labeled graphs induced by vertex languages comparison. An effective method of checking this relation is developed. Properties of vertices maximal by this relation are investigated. It is shown that dominating vertices form a connected component of the graph.

Introduction

Labeled graphs are the most widely used structures in computer science for describing and modeling a variety of computational processes. The most studied in this context are finite oriented graphs with labeled edges (also known as labeled transition systems (LTS) [1], weighted automata [2], finite state automata).

Thus paper is devoted to the study of oriented graphs with labeled vertices which can be seen as dual class in relation to the LTSs. There are many examples where computational processes can be represented more naturally into vertex-labeled graphs than into edge-labeled graphs (in programming [3], robotics [4], model checking [5]).

Languages defined by vertex-labeled graphs have already been studied in the past. In particular the characterization of languages that can be

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represented in different types of vertex-labeled graphs was given in [6], the problem of vertex minimization in that ones was studied in [11], an algebra of vertex-labeled graphs was introduced in [8].

In this work we introduce and study the structure of strongly deterministic vertex-labeled graphs i.e. graphs in which all vertices in neighborhood of every their vertex have different labels. On the vertices of the graph we introduce a domination relation: a vertex v_1 dominates v_2 if the language representable in the graph by v_1 includes the language representable by v_2. This relation generates an indistinguishability relation when this languages are equal.

The paper is organized as follows. In Section 1 we provide the main definition. Un Section 2 we study the structure of the indistinguishability relation. In Section 3 an effective method of construction of the domination relation is developed. Properties of vertices maximal by this relation are investigated. It is shown that maximal by this relation vertices form a connected component of the graph.

1. Basic definitions

A quadruple $G = (V, E, M, \mu)$, consists of a finite set V of vertices, a finite set $E \subseteq V \times V$ of edges, a finite set M of labels, and a mapping $\mu : G \to M$, we will call finite vertex labeled graph (or labeled graph). Let $|V| = n$ and $|M| = m$. The set of vertices adjacent to a vertex $v \in V$ is called the open neighborhood of v and denoted by Γ_v. The set $\Gamma_v \cup \{v\}$ is called closed neighborhood of v and denoted by $\Gamma_{(v)}$. A labeled graph G is said to be deterministic (or D-graph) if all vertices in open neighborhood of every its vertex have different labels. A D-graph G is said to be strongly deterministic (or SD-graph) if all vertices in closed neighborhood of every its vertex have different labels. A SD-graph G is said to be symmetric strongly deterministic (or SSD-graph) if adjacency relation on its vertices is symmetric.

The sequence of vertex labels $w = \mu(g_1) \ldots \mu(g_k)$ corresponding to the path $g_1 \ldots g_k$ in G is said to be word of length $\mathrm{d}(w) = k$ defined by vertex g_1. The set of all words defined by vertex g is said to be the language L_g. The inversion of the word $w = x_1 x_2 \ldots x_k$ is called the word $w^{rev} = x_k \ldots x_2 x_1$. With every subset of vertices $Q \subseteq V$ we will associate the vertex languages union $L_Q = \bigcup_{v \in Q} L_v$.

Let M^* be the set of all finite words over M include empty word, M^+ be the set of all non-empty finite words over M. The partial binary operations $\star : V \times M^+ \to 2^V$ is defined by follows: for every vertex $v \in V$ and every word $w \in M^+$ by $v \star w$ denote the set of all vertices $s \in V$ such that there is a path from v to s labeled by w. The partial binary operation $\circ$ of merging two words is defined on the set M^+ by follows:

$$w_1 x \circ y w_2 = \begin{cases} w_1 x w_2, & \text{if } x = y; \\ \text{undefined}, & \text{otherwise} \end{cases}$$

for any $w_1, w_2 \in M^+$ and any $x, y \in M$.

For any $U, W \subseteq M^+$ we define $U \circ W = \{u \circ w | u \in U \wedge w \in W\}$.

The partial binary operation inverse to the merging is defined by follows

$$u \setminus w = \begin{cases} w', & \text{if } w = u \circ w'; \\ \text{undefined}, & \text{otherwise} \end{cases}$$

for any $u, w \in M^+$. This operation will be extended on power set of M^+ by follows: let $U, W \subset M^+$ and $u \in M^+$ then $u \setminus W = \bigcup_{w \in W} (u \setminus w)$ and $U \setminus W = \bigcup_{u \in U} (u \setminus W)$.

All undefined notions are conventional and can be found for example in [9, 10].

2. Indistinguishability of vertices

With the purpose of analyzing the D-graphs let us introduce the following relation on vertices induced by their languages. We say that vertices $v, s \in V$ are indistinguishable and write $(g, h) \in \varepsilon$ if $L_g = L_h$ [11]. The indistinguishability relation ε is reflexive, symmetric and transitive that there is an equivalence

Theorem 1. *Let G be D-graph then equivalence ε is right stable relative to $\star$.*

Proof. Right stability of ε relative to $\star$ means if vertices $(v, s) \in \varepsilon$ then for any word $w \in L_v \cap L_s$ vertices $(v \star w, s \star w) \in \varepsilon$. We begin by proving that for any vertex $v \in V$ and any word $w \in M^+$ takes place $|v \star w| = 1$ if $w \in L_g$ and $|g \star w| = 0$ otherwise. For all words of length 1 the proof is trivial. For all words of length 2 the proof follows from definition of

D-graph. Notice that any word uniquely describes as merging of its two-letters subwords. Therefore the proof for all words is by induction. Thus we have that vertices $v' = v \star w$ and $s' = s \star w$ are uniquely defined. From this we obtain $L_{v'} = w \backslash L_v$, $L_{s'} = w \backslash L_s$ and $L_{v'} = L_{s'}$ that is $(v', s') \in \varepsilon$. □

With the purpose of analyzing the relation ε let us consider k-indistinguishability of vertices. We will denote by L_v^k the sublanguage of language L_v consists of all words $w \in L_v$ and length w less or equal to some positive integer k. We say that vertices $v, s \in V$ are k-indistinguishable and write $(v, s) \in \varepsilon_k$ if $L_g^k = L_h^k$. It is clear that relation ε_k is an equivalence.

Let $\widetilde{\varepsilon}_i$ be the sequence of relations ε_1, ε_2, ... [7]. It is monotonically decreasing i.e. $\varepsilon_1 \supseteq \varepsilon_2 \supseteq \ldots \supseteq \varepsilon_i \supseteq \ldots \supseteq \varepsilon$. The sequence $\widetilde{\varepsilon}_i$ is said to be stable on step k if from $\varepsilon_k = \varepsilon_{k+1}$ always follows $\varepsilon_k = \varepsilon$.

The following theorem provides a criterion for indistinguishability of D-graph vertices.

Theorem 2. *Let G be D-graph then $\varepsilon = \varepsilon_k$ for some k less or equal to $n - m + 1$.*

Proof. Let us prove that stabilization of the sequence $\widetilde{\varepsilon}_i$ on D-graphs be remained i.e. $\varepsilon_{i+1} = \varepsilon_{i+2} = \ldots = \varepsilon$ follows from $\varepsilon_i = \varepsilon_{i+1}$ for some i. Suppose the vertices $v, s \in V$ satisfies $(v, s) \in \varepsilon_{i+1}$ and $(v, s) \notin \varepsilon_{i+2}$. Hence there exists the word $xyw \in L_v \setminus L_s$ and $\mathrm{d}(xyw) = i+2$, $x = \mu(g) = \mu(h)$, $x, y \in M$, $w \in M^*$. Theorem 1 shows that there exists unique vertices $v', s' \in V$ and $v' = v \star xy$, $s' = s \star xy$. From $(v, s) \in \varepsilon_{i+1}$ and $\varepsilon_i = \varepsilon_{i+1}$ it follows that $(v', s') \in \varepsilon_{i+1}$.

Every ε_k defines a vertex partition π_k on graph G.

Let $|\pi_k|$ denote a number of classes of partition π_k. Suppose $|\pi_i| \neq |\pi_{i+1}|$ holds for all $i \leqslant n - m + 1$. It is clear that $|\pi_1| \geqslant m$. By assumption $|\pi_2| \geqslant m + 1$. We obtain $|\pi_i| \geqslant m + i - 1$ by induction on i up to and including $n - m + 1$. The equality $i = n - m + 1$ implies that $|\pi_{n-m+1}| = n$ i.e. every class consists of single vertex. Further partition is impossible. Hence $\pi_{n-m+1} = \pi_{n-m+2}$.

From what has already been proved it follows that distinguishability of two different vertices of some D-graph can be proved by the word of length $n - m + 1$.

We proceed to show that this estimate is accessible in general case. Consider graph with n vertices on figure1. Let $n = lm + r$ where l, r are

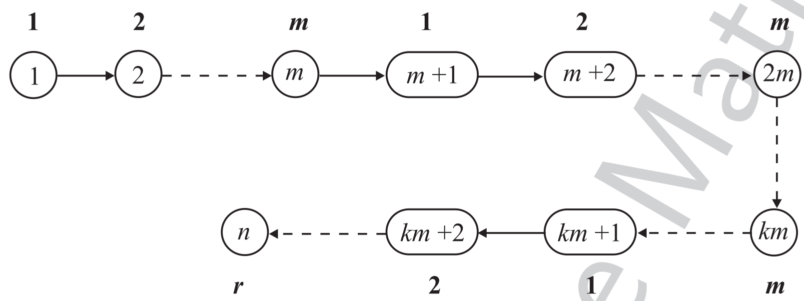

Figure 1.

positive integer and $l < m$. The labeling mapping satisfies the equality $(g_i) = j$ where $g = im + j$, $0 \leqslant i < l$, $1 \leqslant j \leqslant m$. The vertices 1 and $m+1$ are distinguished by the word $(\mathbf{12\ldots m})^{l-1}\mathbf{123\ldots r(r+1)}$ belonging to $L_1 - L_{(m+1)}$ and are indistinguished by all words of lesser length. It is clear that the same graph can be constructed for any positive integer n, $m \leqslant n$.

This completes the proof of Theorem 2. □

The Theorem 2 shows that length estimate of the shortest word distinguishing two vertices of D-graph coincides with upper length estimate of the shortest word distinguishing two states of finite completely defined deterministic Moore automaton [12].

The quotient-graph $G/_\varepsilon$ of graph G by equivalence ε will be called reduced graph and procedure of $G/_\varepsilon$ construction will be called the reduction of G. An algorithm of reduction can be found in [7].

3. Domination of vertices

With the purpose of analyzing the SSD-graphs let us introduce the following relation on the vertices. We say that vertex $v \in V$ dominates vertex $s \in V$ and write $(v, s) \in \varkappa$ if $L_s \subseteq L_v$. The domination relation $\varkappa$ is reflexive and transitive but antisymmetric in general case. Thus $\varkappa$ is an preorder. It is clear that $\varkappa \cap \varkappa^{-1} = \varepsilon$. Let $\varkappa$ be extended on power set of V as follows: if $L_Q \subseteq L_S$ then $(S, Q) \in \varkappa^*$ where $S, Q \subseteq V$.

The following theorem provides a property differing SSD-graphs from D-graphs and finite partial deterministic Mealy automata.

Theorem 3. *Let G be SSD-graph, $H \subset V$ and $v \in V \setminus H$. Then $(H, \{v\}) \in \varkappa^*$ if and only if there exists $s \in H$ and $(s, v) \in \varkappa$.*

Proof. It is clear that $(s, v) \in \varkappa$ implies $(H, \{v\}) \in \varkappa^*$. Let $(H, \{v\}) \in \varkappa^*$ and $(s, v) \notin \varkappa$ holds for any vertex $s \in H$. Suppose $H' = \{s_1, \ldots, s_k\}$ is smallest set that $H' \subseteq H$ and $(H', \{v\}) \in \varkappa^*$. Then there exists the words w_1, ..., w_k that is $w_i \in L_v \setminus L_{s_i}$ where $1 \leqslant i \leqslant k$. Consider the word $w = w_1 \circ w_1^{rev} \circ \ldots \circ w_i \circ w_i^{rev} \circ \ldots \circ w_{k-1} \circ w_{k-1}^{rev} \circ w^k$. Notice that the equality $q \star (u \circ u^{rev}) = q$ holds for any vertex $q \in V$ and any word $u \in L_q$ by definition of SSD-graph. Therefore $w \in L_v$. Let the words w_1, ..., $w_{i-1} \in L_{s_i}$, $1 < i \leqslant k$, then $s_i \star \left(w_1 \circ w_1^{rev} \circ \ldots \circ w_{i-1} \circ w_{i-1}^{rev}\right) = s_i$ and $w_1 \circ w_1^{rev} \circ \ldots \circ w_{i-1} \circ w_{i-1}^{rev} \in L_{s_i}$. Since $w_i \notin L_{h_i}$ it follows that $w_1 \circ w_1^{rev} \circ \ldots \circ w_{i-1} \circ w_{i-1}^{rev} \circ w_i \notin L_{h_i}$.

According to the above the word w is not defined by any vertex of H' i.e. $w \in L_v \setminus L_{H'}$ and $(H', \{v\}) \notin \varkappa^*$. This completes the proof of Theorem 3. □

Let us show that the statement similar to the theorem 3 not holds for D-graph in general case. Let us consider the D-graph on figure 2. Here $(\{1, 3\}, \{5\}) \in \varkappa^*$, $(\{1, 5\}, \{3\}) \in \varkappa^*$, $(\{3, 5\}, \{1\}) \in \varkappa^*$. But any pair of different vertices from the set $\{1, 2, 3\}$ is not an element of $\varkappa$.

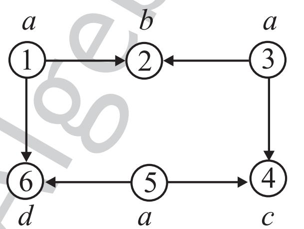

Figure 2.

Note that a relation similar to domination relation can be defined on the set of states of finite partial deterministic Mealy automaton. Let us show that the theorem similar to the theorem 3 not holds for these automata in general case. Let us consider the Mealy automaton on figure 3. Let λ_s be the set of all input-output words generated by the state s [?]. It is clear that $\lambda_1 \subset \lambda_4 \cup \lambda_5$, but $\lambda_1 \not\subseteq \lambda_t$ for $t \in \{2, 3, 4, 5\}$.

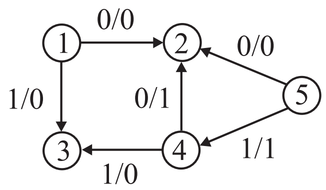

FIGURE 3.

With the purpose of analyzing the relation $\varkappa$ let us consider k-domination relation. We say that vertex $v \in V$ k-dominate vertex $s \in V$ and write $(v, s) \in \varkappa_k$ if $L_s^k \subseteq L_v^k$. It is clear that relation $\varkappa_k$ is an preorder.

Let $\widetilde{\varkappa}_i$ be the sequence of relations $\varkappa_1, \varkappa_2, \ldots, \varkappa_i, \ldots$. By definition of k-dominance this sequence is monotonically decreasing i.e. $\varkappa_1 \supseteq \varkappa_2 \supseteq \ldots \supseteq \varkappa_i \supseteq \ldots \supseteq \varkappa$.

Now we will show that if sequence $\widetilde{\varkappa}_i$ become stable for some i then stabilization remain for all $k \geqslant i$ i.e. the equality $\varkappa_i = \varkappa_{i+1}$ implies equalities $\varkappa_{i+1} = \varkappa_{i+2} = \ldots = \varkappa$. Suppose there exists vertices $v, s \in V$ labeled by the same label $x \in M$ that is $(v, s) \in \varkappa_{i+1}$ and $(v, s) \notin \varkappa_{i+2}$. Then there exists the word $xyw \in L_s - L_v$ that is $\mathrm{d}(xyw) = i + 2$, $y \in M$ and $w \in M^*$. Let $v' = v \star xy$ and $s' = s \star xy$ then $yw \in L_{s'} - L_{v'}$, $\mathrm{d}(yw) = i + 1$, i.e. $(v', s') \notin \varkappa_{i+1}$. This contradiction proofs proposition.

To construct domination relation we use pair-graph introduced in [7].

Let us define a pair-graph of SSD-graph $G = (V, E, M, \mu)$ as a quadruple $P_G = (V_P, E_P, M, \mu_P)$ where $V_P \subseteq \{V \cup \varnothing\} \times \{V \cup \varnothing\}$, $E_P \subseteq V_P \times V_P$ and

1) $V_P = V_P' \cup V_P'' \cup V_P'''$ where $V_P' = \{(v, s) \,|\, v, s \in V \wedge \mu(v) = \mu(s)\}$, $V_P'' = \{(v, \varnothing) \,|\, v \in V\}$, $V_P''' = \{(\varnothing, s) \,|\, s \in V\}$;

2) $\mu_P(v, s) = \mu(v) = \mu(s)$;

3) E_P consists of all edges $e_P = (v_P, s_P)$ that is $g_P \in V_P'$ and if $\mu_P(g_P)\, \mu_P(h_P) = xy$ then $\mathrm{pr}_1\, g_P \star xy = \mathrm{pr}_1\, h_P$, $\mathrm{pr}_2\, g_P \star xy = \mathrm{pr}_2\, h_P$.

In [7] it is shown that the length of shortest path (if it exists) from any vertex $v_P \in V_P'$ to some vertex $s_P \in V_P'' \cup V_P'''$ does not exceed $n^2 + 1$. Thus for checking of the domination between vertices $v, s \in V$ is sufficient to look through all leafs accessible from the vertex $(v, s) \in V_{P'}$ by simple pathes of length less or equal to $n^2 + 1$. It is possible the following variants:

1) if all leafs accessible from the vertex $(v, s) \in V_P'$ are on the form $(\varnothing, s')$ where $s' \in V$ then $(s, v) \in \varkappa$;

2) if all leafs accessible from the vertex $(v, s) \in V_P'$ are on the form $(v', \varnothing)$ where $v' \in V$ then $(v, s) \in \varkappa$;

3) if among all accessible from the vertex $(v, s) \in V_P'$ are vertices of both forms $(\varnothing, s')$ and $(v', \varnothing)$ where $v', s' \in V$ then v and s are not comparable;

4) if no leafs accessible from the vertex $(v, s) \in V_P'$ then $(v, s) \in \varepsilon$ i.e. $(v, s) \in \varkappa$ and $(s, v) \in \varkappa$.

Thus we have the following theorem.

Theorem 4. *Let G be SSD-graph then $\varkappa = \varkappa_k$ for some positive integer k less or equal to $n^2 + 1$.*

It is easy to see that $O\left(n^4\right)$ steps is sufficient for finding all vertices accessible from V_P' (when using the known algorithms of search on the graph [13]).

For the purpose of analyzing domination relation let us consider the vertices maximal by $\varkappa$.

A vertex $v \in V$ is called dominant by $\varkappa$ in graph G if for all $s \in V$ $(s, v) \in \varkappa$ implies $(v, s) \in \varkappa$. We will denote by V^{dom} the set of all dominant vertices. The following result shows that the neighborhood of every dominant vertex consists of only dominant vertices.

Theorem 5. *If $v \in V^{\mathrm{dom}}$, then $O_v \subseteq V^{\mathrm{dom}}$.*

Proof. Let the vertex $v \in V^{\mathrm{dom}}$ and there exists the vertex $v' \in O_v$ that is $v' \notin V^{\mathrm{dom}}$. Then by Theorem 3 there exists a vertex $s \in V^{\mathrm{dom}}$ dominates v'. It is clear that $\mu\left(v'\right)\mu(v) \circ L_v \subseteq L_{v'}$. Hence $\mu\left(v'\right)\mu(v) \circ L_v \subseteq L_s$. Let $s \star \mu(v)\mu\left(v'\right) = s'$ then $L_v \subseteq L_{s'}$ and s' dominates v. If v dominates s' then $L_v = L_{s'}$ and $\mu(v)\mu\left(v'\right) \circ L_{v'} = \mu(v)\mu\left(v'\right) \circ L_s$ by definition of SSD-graph. Hence $L_{v'} = L_s$ and $v' \in V^{\mathrm{dom}}$. If v not dominates s' then $v \notin V^{\mathrm{dom}}$ which is impossible. □

Let us show that statement similar to Theorem 5 not holds for SD-graphs. Let us consider the strongly connected SD-graph on figure 4.

Here $2 \in V^{\mathrm{dom}}$ as single vertex labeled by c. It is ease to check that $(1, 3) \in \varkappa$. Hence $3 \notin V^{\mathrm{dom}}$ in spite of $3 \in O_{(2)}$.

Theorem 5 now shows that if some vertex of connected component G' of SSD-graph G belongs to V^{dom} then all vertices of this component belong to V^{dom}. Therefore to any SSD-graph G corresponds its subgraph $G^{\mathrm{dom}} = \left(V^{\mathrm{dom}}, E^{\mathrm{dom}}, M, \mu\right)$ generated by all dominant vertices where

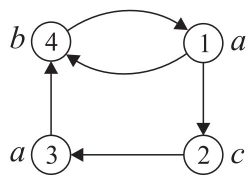

FIGURE 4.

E^{dom} is restriction of E on V^{dom}. The subgraph G^{dom} we will called dominant subgraph. From the above it follows that dominant subgraph consists of all connected components of SSD-graph that contain dominant vertices. The graph isomorphic to its dominant subgraph we will called dominant graph. It is clear that every connected SSD-graph isomorphic to its dominant subgraph. The algorithm of dominant subgraph construction involves finding the set V^{dom} by means of pair-graph and elimination of all connected components with non dominant vertices. As already been shown above the time complexity of $\varkappa$ construction by pair-graph method is $O\left(n^4\right)$. Then $O\left(n^4\right)$ steps is sufficient to construct the dominant subgraph of any SSD-graph.

Note that if dominant graph is reduced by ε then for any different vertices $v, s \in V$ simultaneously holds $L_v \setminus L_s \neq \varnothing$ and $L_s \setminus L_v \neq \varnothing$. Therefore for any $v \in V$ the set $L_v \setminus \bigcup\limits_{s \neq v} L_s$ is nonempty by Theorem 3.

From the above it follows that we can define two procedures on SSD-graphs: reduction of the graph G by ε and finding the dominant by $\varkappa$ subgraph of G. We will show that these procedures can be performed in any order.

Theorem 6. $\left(G/_{\varepsilon}\right)^{\mathrm{dom}} \cong \left(G^{\mathrm{dom}}\right)/_{\varepsilon}$.

Proof. Let us denote by $V^{\overline{\mathrm{dom}}}$ the set of all non dominant vertices of the graph G and by $G^{\overline{\mathrm{dom}}}$ the subgraph generated by this vertices.

Let us apply the reduction algorithm from [7] to the graph G. As the every pair of equivalent vertices belongs only one of the sets V^{dom} or $V^{\overline{\mathrm{dom}}}$ then reduced graph can be in the form of direct sum of the graphs $\left(G^{\mathrm{dom}}\right)/_{\varepsilon} + \left(G^{\overline{\mathrm{dom}}}\right)/_{\varepsilon}$. Let us apply to obtained graph the algorithm of finding the dominant subgraph i.e. eliminate the subgraph $\left(G^{\overline{\mathrm{dom}}}\right)/_{\varepsilon}$. The result of the sequential application of algorithms of reduction and finding dominant subgraph is the graph $\left(G^{\mathrm{dom}}\right)/_{\varepsilon}$.

Let us apply to the graph G the algorithm of finding dominant subgraph and obtain the graph G^{dom}. Let us apply to the graph G^{dom} the reduction algorithm and obtain the graph $\left(G^{\text{dom}}\right)/_{\varepsilon}$. This completes the proof of Theorem 6. $\square$

From idempotency of the graph reduction and the finding of dominant subgraph next statement follows. Let G be SSD-graph and o_1, o_2, ..., o_k be a set of procedures each of which is either reduction or finding subgraph G^{dom}. Hence if exists at least one reduction procedure among the set o_1, o_2, ..., o_k then the result of the sequential application of the procedures o_1, o_2, ..., o_k to the graph G is the graph $(G/_{\varepsilon})^{\text{dom}}$.

Conclusion

In this paper for the purpose of analyzing the structure of SSD-graphs the relation of dominance on the vertices of labeled graph is introduced. An effective method of checking this relation is developed. Properties of vertices extremal in this relation are investigated. It is shown that dominating vertices form a connected component of the current graph. The obtained results serve as the basis of the synthesis of distinguishing and checking experiments with labeled graphs. Note the fact that the analysis of the graphs provided by methods similar to methods of automata theory. This methods was effectively extended to graph systems that are not finite automata but in some sense are automata-like systems.

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