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Preradical and kernel functors over categories of $S-$acts

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Abstract. We concider the big lattices of preradicals and kernel functors over some cathegories of centered $S-$acts, where S is monoid whit zero. We prove that those big lattices are two elements if and only if monoid $S-$ is groups with zero. A subset of a Rees generated pretorsion theory is a subquantale of quantale of pretorsion theory.

Introduction

The paper is devoted to our own vision of ways of development of the theory of preradicals and hereditary preradicals, the latter we call kernel functors, as well as Goldman. However, more important of the preradicals and kernel functors call radicals and tortions theories, and they are studied by many authors in different categories. General constructions and facts of the preradical theories in the category of modules will serve as a basic references and models or, if you want, standards in studying of the corresponding objects in the context of the category of polygons we must approximate to. But we were forced to ask a lot of questions, which are mostly open (at least, the authors don't know the answers, and couldn't find it in accessible literature). Further, we don't know any article, devoted to the theory of the preradicals in polygons. One of the most simple and natural terms, that works in a lot of spheres of mathematics is a concept "acts over groups". Basic technical means, connected with this concept, are studied even by first year students. Corresponding G-sets are basic in a theory of kogomology groups building. Historically acts over groups were first investigated by P. Ruffini, A. Cayley, S. Lie. (see S.Lie Die Grundlagen

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fur die Theorie de unendlichen kontinujerlichen Transfomatinsgruppen, I und II. Leipzig, Derichte,3,316-362,353-93, 1891). More general term act over groups or monoid is not less important, but more difficult, as far as theory of semigroup is more difficult and less developed by comparison to the theory of groups. By the way, the word "demigroup" was first used by Seguier (L'abbe of J.A.Seguier) in 1904 in his book "Elements of de la theorie des groups abstractes", that was published in Paris. In 1905 an article was published , fully devoted to semigroups, where he cites Seguier. In 1916 O. J. Schmidt introduced the term "semigroup" on a regular basis in his Russian-language book "Abstract Group Theory". But Schmidt in his determination of semigroup means the semigroups which are cancellative from both sides. Acts over semigroups Lev Anatolievich Skornyakov named "polygon" and we also will adhere to this term. It is used by all of Russian-language mathematics, but this is not the only concept in the articles, devoted to acts over semigroups. With the purpose of illumination of some moments in history of origin and development of theory of grounds it is necessary to indicate terminology, which was used by other scientists who have played an important role in the development of this theory. Moreover, that the category of polygons over monoid is equal to the category of representation of this monoid. Notice, that in case to inverse monoid, this category is a topos that substantially distinguishes it from the categories of the acts over the associative ring. In fact the category of acts over any ring can not be a topos. In particular category of polygons is not Abelian. It mostly not additive, therefore methods of investigating of polygons essentially differ from methods that are used in theory of rings and acts, whether in any other section of algebra. Thus, it is impossible to make equal most of theorems of the ring and acts theory to the theory of polygons, therefore much of this facts have natural analogues in the polygon theory.

To sum up, it can be said that the theory of the acts is a generalization of a set theory, as it acts theory over rings. So it shows corporate properties, which are presented in both categories. This thesis emphasizes results, obtained by L. A. Skornjakov. The results are related to axiomatization classes of all acts, that are some analogue of axiomatic set theory, which was proposed by Louwer.

General results of that theory now are collected in the monograph of M. Kilp, U. Knauer, A.V. Mikhalev. We refer to this monograph, where it is possible quickly to acquaint with the theory, to understand the special features and the attractiveness of the theory, especially for those mathematics, who work in a sphere of acts theory.

The concept of quantale [7] goes back to 1920's, when W. Krull, followed by R. P. Dilworth and M.Ward, considered a lattice of ideals equipped with multiplication. The term "quantale" itself (short for "quantum locale") was suggested by C.J. Mulvey. A quantale is a complete lattice L satisfying the law $a \cdot (\bigvee_{i\in I} b_i) = \bigvee_{i\in I}(a \cdot b_i)$ for all $a, b_i \in L$, where I is an index set. Note, that in this definition we can substitute the meets by the joins, so that we obtain the dual quantale with respect to meets.

It is well known that the set of all ideals of any ring is a complete lattice on which an operation of multiplication of ideals is defined. It satisfies an infinite distributive law, so the lattice of all ideals of the ring is a quantale. By analogy, the set of all preradical filters of left ideals of an arbitrary ring is a complete lattice. Moreover, the operation of Gabriel multiplication of preradical filters is defined. Hence, the lattice of preradical filters is, in fact, a quantale with respect to meets, see [1]. We prove that some analog results are valid in cass of categories of acts.

Preliminaries

Throughout the paper, all monoids are assumed to have zero element.

Analogue of the notion of ring and module for cases of semigroups and monoids are acts. Remark that A. K. Syshkewich was the first, who made systematic research of acts, but he used another term. His dissertation is named as "The theory of action as generalized group theory" (Russian), 1922).

Let's give some basic definitions.

Definition 1. *Let S be a monoid and $A \neq \emptyset$ be a set. If there exist mapping $\mu : S \times A \to A$, such that*

1. $1a = a$;
2. $(st)a = s(ta)$ *for all* $a \in A$, $s, t \in S$,

we call A a left $S-$act.

Recall that instead of the term "act" sometimes are used another notions: set, operand, action, system, automat.

All acts are unitary and centered left $S-$act.

Definition 2. *Let A and B be two left $S-$acts. Recall that the mapping $f : A \to B$ is called homomorphism if $f(sa) = sf(a)$ for all $s \in S$ and $a \in A$.*

The set of all S-homomorphisms from ${}_SA$ into ${}_SB$ will be denoted by $Hom({}_SA,{}_SB)$ or sometimes by $Hom_S(A, B)$. We consider category of left $S-$acts and their homomorphisms and denote it by $S - Act$.

Definition 3. *An S-act is called a multiplication if for every subact $B \subset A$ exist some ideal I of S such that $IA = B$.*

Any subact $B \subset A$ defines Rees congruence ρ_B on A, by setting $a\rho_B b$ if $a, b \in B$ or $a = b$. We denote the resulting factor act by A/B.

In modern researches of acts different approaches often appear, that are related in consideration of different category of acts. Except the category of all acts, we research also two categories. We'll recall to the first construction of Lex-Wiegandt category of acts(see [4]) and then to the second category of multiplication acts. Denoted by $S - LWAct$ and $S - MAct$, respective categorys.

Definition 4. *An $S-$act M is called a simple, if it contain only trivial subacts.*

Definition 5. *An $S-$act M is called a congruence simple, if it has only trivial congruences, i.e. $Con(M) = \{\Delta, \nabla\}$, where $\Delta = \{(a, b) \in M \times M \,|a = b\}$ and $\nabla = \{(a, b) \in M \times M \,|a, b \in M\}$.*

Definition 6. *Let I be two-sided ideal of semigroup S. Left $S-$act M is called $I-$divisible, if $IM = M$. Notion of $\rho-$divisible act is introduced in the same way, i.e. if $(\rho)(M \times M) = (M \times M)$.*

Definition 7. *A left socle of left $S-$act M is a union of all simple subacts in M. This subact is denoted by $Soc(M)$. Thus $Soc(M) = \bigcup\limits_i N_i$, where N_i - run over all simple left subacts in M.*

Insofar any two simple subacts of $S-$act M either don't intersect or coincide, this union is disjunctive union, i.e. $Soc(M) = \coprod\limits_i N_i$.

Definition 8. *By $Soc_{Con}(M)$ we'll denote a union of all congruence simple subacts in M. It means that $Soc_{Con}(M) = \bigcup\limits_i N_i$, where N_i - congruence simple subact in M.*

If any homomorphism between congruence simple subacts, then $f(M)$ $-$ congruence simple subact or $f(M)$ is equal to zero. Then $\sigma(M) = \bigcup M_i$ and $\sigma(N) = \bigcup N_i$. Insofar $f(\bigcup M_i) = \bigcup f(M_i)$, it induce mapping $f' : \sigma(M) \to \sigma(N)$, that is restriction of f on $\sigma(M)$.

Note, that direct product of $S-$acts is not product in category $\mathcal{C}$ because projections on components are not Rees's quotient-mappings. Besides, product of two $S-$acts don't necessary exists.

Definition 9. *A $S-$act N is called essential in M if $K \cap N \neq 0$ for every no trivial subact K of M.*

1. Preradicals and kernel functors in $S-$Act

Definition 10. *A* preradical *in the category $S - Act$ is a functor*

$$\sigma : S - Act \to S - Act$$

such that:

1. $\sigma(M)$ is subact in M for each $M \in S - Act$;

2. For each homomorphism $f : M \to N$, the diagram

$$\begin{tikzcd} \sigma(M) \arrow[r, hook] \arrow[d] & M \arrow[d, "f"] \\ \sigma(N) \arrow[r, hook] & N \end{tikzcd}$$

is commutative.

Denote by $S - pr$ the complete big lattice of all preradicals in $S - Act$. There is a natural partial ordering in $S - pr$ give by $\sigma \preceq \tau$ if $\sigma(M) \subseteq \tau(M)$ for each $M \in S - Act$.

The most natural examples of the preradicals are the functors $Soc(M)$ and $Soc_{Con}(M)$.

Notice that the preradicals are the subfunctors of identity functor.

Definition 11. *Let S be a monoid and M be a left $S-$act. For all $m \in M$ define a set*

$$Ann(m) = \{(a, b) \in S \times S | am = bm\}.$$

Then $Ann(m)$ call annihilator of element m and annihilator of M is $Ann(M) = \bigcap\limits_{m \in M} Ann(m)$.

Obviously, $Ann(M)$ is a left congruence on S.

Let S be a commutative monoid and A be a left $S-$act. Denoted by $Z(A) = \{a \in A | Ann(a)$ is a essential congruence on S $\}$. Then $Z(A)$ is singular part of $S-$act A.

Lemma 1. *Let S be a commutative monoid and A be a left $S-$act. Then $Z(A)$ is preradical.*

Proof. If $a \in Z(A)$ then show that $sa \in Z(A)$ for all $s \in S$. Since, $Ann(a)$ is a essential congruence on S and S is a commutative monoid, then $sta = sra$ and $tsa = rsa$ for all $s \in S$, follows that $sa \in Z(A)$.

We induce mapping $f' : Z(M) \to Z(N)$, that is restriction of f on $Z(M)$. □

Definition 12. *A left $S-$act A call prime if $Ann(A) = Ann(B)$ for all no trivial subacts B of A.*

Definition 13. *A left $S-$act A call prime if for all congruence ρ on S such that for same $a \in A$ implies $sa = ta$ for all $(s,t) \in \rho$ then $\rho \subseteq Ann(A)$ or $a = 0$.*

Lemma 2. *Let S be a monoid and A be a left $S-$act. Then the following conditions equivalent:*

1. *$Ann(A) = Ann(B)$ for all no trivial subacts B of A.;*

2. *For all congruence ρ on S such that for same $a \in A$ implies $sa = ta$ for all $(s,t) \in \rho$ then $\rho \subseteq Ann(A)$ or $a = 0$.,*

Proof. Let hold condition 2, but no condition 1. Then exist no trivial subact B of A and $Ann(A) \neq Ann(B)$. We have congruence σ on S and exist $b \in B$ and $b \neq 0$ such that $sb \neq tb$ for all $(s,t) \in \sigma$. Hence, A is not prime by condition 2. A contradiction.

Let A is $S-$act and exists some congruence ρ on S such that for same $a \in A$ implies $sa = ta$ for all $(s,t) \in \rho$. We need to show that $\rho \subseteq Ann(A)$. Suppose that exist $s_1, t_1 \in S$ and $b \in A$ such that $(s_1, t_1) \in \rho$ but $s_1 b \neq t_1 b$. Then $B = Sa$ is subact of A. Hence, $Ann(A) \neq Ann(B)$. A contradiction. Therefore, $\rho \subseteq Ann(A)$.

□

Definition 14. *An S-act is called completely reducible if it is a disjoint union of simple subacts.*

Proposition 1. *(see [3] p. 75) All acts over a monoid S are completely reducible if and only if S is a group.*

Theorem 1. *Let S commutative monoid. Then the following statements hold:*

1. *In the category $S-Act$ all preradicals are trivial.*

2. *In the category $S-LWAct$ all preradicals are trivial.*

3. *In the category $S-MAct$ all preradicals are trivial.*

4. *S is group with zero.*

Proof. (1) $\Rightarrow$ (2). The category $S-LWAct$ is subcategory of the categories $S-Act$. Thus in the category $S-LWact$ all preradicals are trivial. (2) $\Rightarrow$ (3). Let in the category $S-MAct$ exist objects A such that $\sigma(A)$ is no trivial. Then this S-act is in the category $S-LWAct$, but all preradicals are trivial in this category. A contradiction. (3) $\Rightarrow$ (4). If in the category $S - MAct$ all preradicals are trivial. Then every S-act A from $S - MAct$ contains only trivial subacts. Hence, monoid S not contains no trivial ideals and S is a group. (4) $\Rightarrow$ (1). If S is a group by Proposition 1: all acts over a monoid S are completely reducible. Then in the category $S - Act$ preradicals for all objects A are trivial. $\square$

There are following classical operations in $S - pr$, namely $\wedge$, $\vee$, $\cdot$ which are defined as follows, for $\sigma, \tau \in S - pr$ and $M \in S - Act$:

$(\sigma \wedge \tau)(M) = \sigma(M) \cap \tau(M)$,

$(\sigma \vee \tau)(M) = \sigma(M) \cup \tau(M)$,

$(\sigma \cdot \tau)(M) = \sigma(\tau(M))$.

The meet $\wedge$ and the join $\vee$ can be defined for arbitrary families $\mathcal{C}$ of preradicals as follows:

$\tau = \wedge\{\sigma \in \mathcal{C}\}$ such that $\tau(M) = \cap\{\sigma(M)|\sigma \in \mathcal{C}\}$

$\mu = \vee\{\sigma \in \mathcal{C}\}$ such that $\mu(M) = \cup\{\sigma(M)|\sigma \in \mathcal{C}\}$

Notice that for each $M \in S - Act$, $\{r(M)\,|r \in \mathcal{C}\}$ is a set.

The operation $\cdot$ is called a *product*. It is well known that $r_1 \cdot r_2 \leq r_1 \wedge r_2 \leq r_1 \vee r_2$.

All these operations are associative and order-preserving.

Definition 15. *(see [12]) The functor $\sigma \in S - pr$ is a left exact preradical if for each short exact sequence*

$$0 \to L \xrightarrow{f} M \xrightarrow{g} N \to 0$$

the sequence

$$0 \to r(L) \xrightarrow{\sigma(f)} r(M) \xrightarrow{\sigma(g)} r(N)$$

is exact.

The functor $\sigma \in S - pr$ is a *radical* if $\sigma(M/\sigma(M)) = 0$ for each $M \in S - Act$.

For any $\sigma \in S - pr$, we will use the following four classes of acts:

$\mathrm{T}_\sigma = \{M \in S - Act\,|\sigma(M) = M\}$;

$\mathrm{F}_\sigma = \{M \in S - Act\,|\sigma(M) = 0\}$;

$\bar{\mathrm{T}}_\sigma = \{\sigma(M)|M \in S - Act\}$;

$\bar{\mathrm{F}}_\sigma = \{M/\sigma(M)|M \in S - Act\}$

Recall that σ is idempotent if and only if $\mathrm{T}_\sigma = \bar{\mathrm{T}}_\sigma$, σ is radical if and only if $\mathrm{F}_\sigma = \bar{\mathrm{F}}_\sigma$. The functror σ is a *left exact preradical* if it is idempotent and its pretorsion class T_σ is closed under taking subacts. The functor σ is a *radical* if and only if it is radical and its pretorsion-free class F_σ is closed under taking quotient acts.

Let $\sigma, \tau, \eta \in S-pr$, $\{\sigma_\alpha\}_\alpha \subseteq S-pr$, $M \in S-Act$. Then the following properties hold:

1. $\sigma \leq \tau \Rightarrow \sigma \vee (\tau \wedge \eta) = \tau \wedge (\sigma \vee \eta)$(Modular law);
2. If $\{\sigma_\alpha\}_\alpha$ is a directed family, then $\tau \wedge (\vee_\alpha \sigma_\alpha) = \vee_\alpha (\tau \wedge \sigma_\alpha)$;
3. $(\wedge_\alpha \sigma_\alpha)\tau = \wedge_\alpha (\sigma_\alpha \tau)$;
4. $(\vee_\alpha \sigma_\alpha)\tau = \vee_\alpha (\sigma_\alpha \tau)$.

The classes of idempotent preradicals are closed under taking arbitrary joins, and the classes of radicals and left exact preradicals are closed under taking arbitrary meets.

A preradical $\sigma : S - Act \to S - Act$ is called a *kernel functor* if for every $M \in S - Act$ and any subact N of M, $\sigma(N) = N \bigcap \sigma(M)$.

For additional information on kernel functors see [2].

Lemma 3. *Let I be two-sided ideal of semigroup S. For every act $M \in S - Act$ put $\sigma(M) = IM$, where IM is essential in M. Then*

1. σ is a kernel functors.

2. σ is a radical.

2. Lattices of pretorsion theories

Definition 16. *A* torsion theory*([5]) τ for the category $S - Act$ is a pair $(\boldsymbol{T},\boldsymbol{F})$ of classes of $S - Act$ satisfying the following conditions:*

1. $Hom_S(T, F) = 0$ for all $T \in \boldsymbol{T}$ and $F \in \boldsymbol{F}$;

2. If $Hom_S(M, F) = 0$ for every $F \in \boldsymbol{F}$ then $M \in \boldsymbol{T}$;

3. If $Hom_S(T, N) = 0$ for all $T \in \boldsymbol{T}$ then $N \in \boldsymbol{F}$.

Definition 17. *A quasi-filter ([11]) of S is defined to be subset $\mathcal{E}$ of $Con(S)$ satisfying the following conditions:*

1. If $\rho \in \mathcal{E}$ and $\rho \subseteq \tau \in Con(S)$, then $\tau \in \mathcal{E}$.

2. $\rho \in \mathcal{E}$ implies $(\rho : s) \in \mathcal{E}$ for every $s \in S$.

3. If $\rho \in \mathcal{E}$ and $\tau \in Con(S)$ such that $(\tau : s), (\tau : t)$ are in $\mathcal{E}$ for every $(s, t) \in \rho \backslash \tau$, then $\tau \in \mathcal{E}$.

Subset $\mathcal{E}$ of $Con(S)$ satisfying 1 and 2 condition are called *preradical quasi-filters* or *preradical filters.*

Remind that a class T is a torsion class for a torsion theory τ if and only if it is closed under quotient acts, direct sums and extensions. A class F is a torsion-free class for τ if and only if F is closed under subacts, direct products and extensions.

The acts in T are called τ-*torsion*, and the ones in F are τ-*torsion-free.*

Since the intersection of an arbitrary family of quasi-filters is a quasi-filter, we may see that the set of all quasi-filters has the structure of a complete lattice, where the meet and the join of quasi-filters are defined in the usual way.

A meet of the preradical filters $\mathcal{F}_1$ and $\mathcal{F}_2$ is the preradical filter $\mathcal{F}_1 \bigwedge \mathcal{F}_2$ which is the intersection of $\mathcal{F}_1$ and $\mathcal{F}_2$.

A join of preradical filters $\mathcal{F}_1$ and $\mathcal{F}_2$ is the least preradical filter $\mathcal{F}_1 \bigvee \mathcal{F}_2$ which contain both $\mathcal{F}_1$ and $\mathcal{F}_2$.

A product of preradical filters $\mathcal{F}_1$ and $\mathcal{F}_2$ is a set $\mathcal{F}_1 \cdot \mathcal{F}_2$ of those left congruence α of S for which there exists left congruence $\beta \in \mathcal{F}_2$ such that $\alpha \subseteq \beta$ and $(\alpha : s), (\alpha : t) \in \mathcal{F}_1$ for all $(s,t) \in \beta$.

We will also need the following fact.

Proposition 2. *If $\mathcal{E}_1$ and $\mathcal{E}_2$ are (preradical) quasi-filters of left congruence on a monoid S , then their product $\mathcal{E}_1 \cdot \mathcal{E}_2$ is a (preradical) quasi-filter of left congruence on a monoid S.*

Proof. We need to show that the filter $\mathcal{E}_1 \cdot \mathcal{E}_2$ satisfies the second condition. Let $\alpha \in \mathcal{E}_1 \cdot \mathcal{E}_2$. Then exists such $\beta \in \mathcal{E}_2$ that $\alpha \subseteq \beta$ and $(\alpha : s), (\alpha : t) \in \mathcal{F}_1$ for all $(s,t) \in \beta$. Then exist such left congruence $\gamma \in \mathcal{E}_2$. Consider the left congruence $\tau = \alpha \bigcap \gamma$ of S. Since $\tau \subseteq \beta \bigcap \gamma$ and $\beta \bigcap \gamma \in \mathcal{E}_2$, the inclusion $(\tau : a) = (\alpha \bigcap \gamma : a) \supseteq (\alpha : a)$ and $(\tau : b) = (\alpha \bigcap \gamma : b) \supseteq (\alpha : b)$, for every $(a,b) \in \beta \bigcap \gamma$, follows that $(\tau : a), (\tau : b) \in \mathcal{E}_2$. It means that $\tau \in \mathcal{E}_1 \cdot \mathcal{E}_2$. □

3. Quantales of quasi-filters

A *quantale* Q is a complete lattice with an associative binary multiplication $*$ satisfying

$$x * \left(\bigvee_{i \in I} x_i \right) = \bigvee_{i \in I} (x * x_i)$$

and

$$\left(\bigvee_{i \in I} x_i \right) * x = \bigvee_{i \in I} (x_i * x)$$

for all $x, x_i \in Q$, $i \in I$, I is a set. By 1 denotes the greatest element of the quantale Q, by 0 is the smallest element of Q. A quantale Q is said to be *unital* if there is an element $u \in Q$ such that $u * a = a * u = a$ for all $a \in Q$.

By a *subquantale* of a quantale Q is meant a subset K closed under joins and multiplication.

Proposition 3. *The set of all quasi-filters of the monoid S forms a quantale with respect to meets.*

Theorem 2. *The set of all preradical quasi-filters of left congruence on a monoid S is a quantale with respect to meets, which is a subquantale of the quantale of all preradical filters of left congruence on a monoid S.*

Proofs are straightforward and left to the reader.

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