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RESEARCH ARTICLE

On some locally soluble infinite dimensional linear groups

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Abstract. The author investigates nonabelian locally soluble linear groups of infinite fundamental dimension and of infinite section p-rank all of whose proper nonabelian subgroups of infinite section p-rank are of finite fundamental dimension. The solubility of these groups is proved.

A group G of all automorphisms of a vector space A over a field F is called a full linear group. This group is denoted by $GL(F, A)$. The subgroups of this group are called linear groups. The group $GL(F, A)$ and its distinct subgroups are the oldest subjects of investigation in Group Theory. If the dimension $dim_F A$ of A over F is finite, then the group $GL(F, A)$ can be identified with the group of non-singular square matrices of dimension $n \times n$ where $n = dim_F A$. Finite dimensional linear groups have played an important role in mathematics.

The study of the subgroups of $GL(F, A)$ in the case when $dim_F A$ is infinite has been limited and requires some additional restrictions. The circumstances here are similar to those that appeared in the early period of development of the Infinite Group Theory. One of the most fruitful approaches here is the application of finiteness conditions to the study of infinite groups. One such restriction on infinite dimensional linear groups is the notion of a finitary linear group. A group G is called finitary if for each element g the subspase $C_A(g)$ has finite codimension in A. Finitary linear groups were investigated by many authors [1, 2]. The papers [3] and [4] have discussed certain other types of finiteness conditions in linear

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groups, analogous to the well-known theories of groups with the minimal and maximal conditions on subgroups.

If H is a subgroup of $GL(F, A)$ then H really acts on the quotient spase $A/C_A(H)$ in a natural way. We denote $dim_F(A/C_A(H))$ by $centdim_F(H)$. If $dim_F(A/C_A(H))$ is finite we shall say that H has finite central dimension; otherwise we shall say that H has infinite central dimension. A group G is therefore finitary linear precisely when each its cyclic subgroup has finite central dimension. The central dimension of a subgroup depends on the vector space on which it is acting. In [5] we studied linear groups of infinite central dimension and infinite p-rank all of whose proper subgroups of infinite p-rank are of finite central dimension. Let H be a subgroup of $GL(F, A)$. We consider the subspase $[H, A]$ of space A, which is generated by the elements $[H, A] =< v(g - 1), g \in H, v \in A >$. The dimension of subspase $[H, A]$ is called the fundamental dimension of group H. The fundamental dimension of group H is denoted by $ddim_F H = dim_F[H, A]$. In [6] we studied linear groups of infinite fundamental dimension and infinite p-rank all of whose proper subgroups of infinite p-rank are of finite fundamental dimension. In this paper the author investigates the nonabelian locally soluble linear groups of infinite fundamental dimension and of infinite p-rank all of whose proper nonabelian subgroups of infinite p-rank are of finite fundamental dimension.

A group G is said to have finite 0-rank (or finite torsion-free rank) $r_0(G) = r$, if G has subnormal series with r infinite cyclic factors, all other factors being periodic. It is well known that the 0-rank is independent of the chosen series. In the case of polycyclic-by-finite groups the 0-rank is simply the Hirsch number. Locally almost soluble groups of finite torsion-free rank have been well studied [7].

Now let p be a prime. A group G has finite section p-rank $r_p(G) = r$, if every elementary abelian p-section of G is finite of order at most p^r and there is an elementary abelian p-section U/V such that $|U/V| = p^r$. In this paper when speaking of the section p-rank we shall assume that p = 0 or that p is a prime.

Later the section p-rank of a group is called p-rank of this group for convenience of the account.

Lemma 1. Let $G \leq GL(F, A)$ be a nonabelian group and suppose that $r_p(G)$ and и $ddim_F G$ are infinite. Suppose that every proper nonabelian subgroup of infinite p-rank has finite fundamental dimension.

(i) If U and V are proper nonabelian subgroups of a group G and $G =< U, V >$ then at least one of U, V has finite p-rank.

(ii) If H is a proper nonabelian subgroup of group G of infinite p-rank then every subgroup of H and every proper subgroup of G containing H

has finite fundamental dimension.

(iii) If K and L are proper subgroups of G containing H, then $< K, L >$ is a proper subgroup of G.

Lemma 2. Let $G \leq GL(F, A)$ be a soluble nonabelian group and suppose that $r_p(G)$ and и $ddim_F G$ are infinite. Suppose that every proper nonabelian subgroup of infinite p-rank has finite fundamental dimension. If H is a proper normal subgroup of infinite p-rank and G/H is finite generated, then G/H is a cyclic q-group for some prime q.

Proof. We consider at first the case of nonabelian subgroup H. Then for every element $g \in G$, $g \notin H$ the subgroup $< H, g >$ is nonabelian. Suppose that $G =< H, S >$ for some finite set S with the property that if R is a proper subset of S, then $G \neq< H, R >$. Suppose that S consists of the elements $x_1, x_2, ..., x_k$. If $k > 1$, then $< H, x_1, x_2, ..., x_{k-1} >$ and $< H, x_k >$ are proper nonabelian subgroups of infinite p-rank. Lemma 1 provides a contradiction. It follows that G/H is cyclic. If G/H is infinite, or if G/H is finite but $|\pi(G/H)| > 1$, then G is a product of two proper nonabelian subgroups, which again gives a contradiction. Hence G/H is a finite cyclic q-group for some prime q.

Let the subgroup H be an abelian now. Let H_1 be a maximal normal abelian subgroup of G, which contains the subgroup H and $C = C_G(H_1)$. Then for every element $g \in G$, $g \notin C$, a subgroup $< C, g >$ is nonabelian. In the case when G/C is not cyclic q-group for some prime q the group G is a product of two proper nonabelian subgroups of infinite p-rank. Lemma 1 provides a contradiction.

We consider the last case, when $G/C =< g > C/C$ is a cyclic q-group for some prime q. Let subgroup C be an abelian. Suppose that a does not contained in the center of group G. Let $M = C/ < a >^G$ and $G_1 = G/ < a >^G$. According to the construction the quotient group M has infinite p-rank. Let T be a periodic part of M. Let $p \geq 0$ and $M \neq T$. Then the quotient group M/T is a non-trivial abelian torsion-free group. We consider the case, when M/T is divisible at first. Since G/C is finite, then M/T is a direct product of G-invariant subgroups of finite p-rank by the theorem 5.9 [8]. If p-rank of a quotient group M/T is infinite then the group G_1 is a product of two proper subgroups $N_1 = S_1/ < a >^G$ and $N_2 = S_2/ < a >^G$ of infinite p-rank such that the subgroups S_1 and S_2 contain the element g. Therefore $G = S_1 S_2$ and G is a product of two proper nonabelian subgroups of infinite p-rank. Lemma 1 provides a contradiction.

Let M/T be indivisible. Then there is a prime r for which the quotient group $(M/T)/(M/T)^r$ is non-trivial. If $r \neq q$ and the quotient group $(G_1/T)/(M/T)^r$ is infinite then by the theorem 5.9 [8] the group G_1 is a product of two proper subgroups $G_2 = L/ < a >^G$ and $G_3 = F/ < a >^G$

of infinite p-rank where the subgroups L and F containe the element g. By the construction the subgroups L and F are nonabelian and $G = LF$. Lemma 1 provides a contradiction. Let $r = q$ and the quotient group $(G_1/T)/(M/T)^r$ be infinite. Then $K = (G_1/T)/(M/T)^r$ is nilpotent and q-rank of K is infinite by lemma 6.34 [9]. K/K' is infinite by lemma 2.22 [9]. Therefore the group G is a product of two proper nonabelian subgroups of infinite p-rank which containe the element g. Lemma 1 provides a contradiction.

Suppose that a quotient group $(M/T)/(M/T)^r$ is finite for any prime r and there are two different prime numbers r_1 and r_2 which are different from q, such that the quotient groups $(M/T)/(M/T)^{r_1}$ and $(M/T)/(M/T)^{r_2}$ are non-trivial. Then $(G_1/T)/(M/T)^{r_1 r_2}$ is an extension of abelian $\{r_1, r_2\}$-group by cyclic q-group. Therefore $G/ < a >^G$ is a product of two proper subgroups of infinite p-rank $L_1/ < a >^G$ and $F_1/ < a >^G$, where subgroups L and F containe the element g. By the construction L_1 and F_1 are nonabelian subgroups and $G = L_1F_1$. Lemma 1 provides a contradiction. If there are no such prime numbers r_1 и r_2, then G_1/T containes a normal divisible abelian torsion-free subgroup of finite index. The result follows by the proof of the case of divisible quotient group M/T.

If the subgroup M is periodic then we shall denote by M_0 the Sylow p'-subgroup of M. A quotient group G_1/M_0 is an extension of abelian p-group of infinite p-rank by cyclic q-group. If M/M_0 is an almost divisible group then let M_1/M_0 be its lower layer. The quotient group G_1/M_1 is an almost divisible quotient group. A subgroup M_1 has infinite p-rank too. By theorem 5.9 [8] the quotient group G_1/M_1 is a product of two proper non-trivial subgroups N_1/M_1 and N_2/M_1, where $N_1 = L_2/ < a >^G$, $N_2 = F_2/ < a >^G$. Subgroups L_2 and F_2 containe the element g. By the construction L_2 and F_2 are nonabelian subgroups of infinite p-rank and $G = L_2F_2$, which again gives a contradiction. We consider the last case where the quotient group G_1/M_0 is not almost divisible group. If a period of M/M_0 is infinite, then we consider the quotient group G_1/M_1 as in the previous case. It has the normal abelian p-group which has infinite p-rank and finite index. The result follows from the proof of the previous case. Let a period of the quotient group M/M_0 be finite. If $q \neq p$ then by the theorem 5.9 [8] M/M_0 is a direct product of infinite number of G-invariant subgroups. Thus the quotient group G_1/M_0 is a product of two proper subgroups of infinite p-rank which containe the element g. Therefore G is a product of two proper nonabelian subgroups of infinite p-rank. Lemma 1 provides a contradiction. If $q = p$ then the quotient group G_1/M_0 is a nilpotent group of infinite p-rank by the lemma 6.34 [9]. $(G_1/M_0)/(G_1/M_0)'$ is infinite by the lemma 2.22 [9]. Thus G is a product of two proper nonabelian subgroups of infinite p-rank, which again gives a contradiction.

In the case of nonabelian subgroup C the quotient group C/H is not cyclic. Let be $G/C =< g > C/C$. Then $< H, g >$ is a proper nonabelian subgroup of group G. Thus G is a product of two proper nonabelian subgroups C and $< H, g >$ of infinite p-rank. Lemma 1 provides a contradiction. The lemma is proved.

Lemma 3. Let $H \leq GL(F, A)$. If H is a locally soluble group of finite fundamental dimension then H is soluble.

Proof. Let $B = [H, A]$, $C = C_H(B)$. Then $dim_F(A/C)$ is finite. The quotient group H/C is soluble by corollary 3.8 [10]. Each element of C acts trivially on every factor of the series $< 0 >\leq B \leq A$. Thus C is abelian. Therefore H is a soluble group. The lemma is proved.

Lemma 4. Let G be a nonabelian locally soluble subgroup of $GL(F, A)$ and G has infimite fundamental dimension and infinite p-rank for some $p \geq 0$. Suppose that every proper nonabelian subgroups of infinite p-rank of G has finite fundamental dimension. If the group G is not soluble then it is perfect.

Proof. Suppose at first that H is a proper normal subgroup of G and the index $|G : H|$ is finite. If H is an abelian subgroup then the group G is soluble. Contradiction. Thus the subgroup H is nonabelian. Therefore $ddim_F(H)$ is finite. Lemma 3 gives a solubility of H. Thus a quotient group G/H is abelian by lemma 2. Therefore G is soluble. Contradiction. Let $G \neq G'$. Thus a quotient group G/G' is divisible. Then G containes a proper normal subgroup H for which $G/H \simeq C_{q^\infty}$ for some prime q. Thus $r_p(H)$ is infinite. If H is nonabelian then H has finite fundamental dimension. In the case of abelian subgroup H there exists its finite nonabelian extension H_1. H_1 is a proper nonabelian subgroup of G. Thus the rank $r_p(H_1)$ is infinite. Therefore H_1 has finite fundamental dimension. Thus H has finite fundamental dimension too. H is soluble by lemma 3. Therefore G is soluble too. Contradiction. The lemma is proved.

Lemma 5. Let G be a nonabelian locally soluble subgroup of $GL(F, A)$ and suppose that G has infinite fundamental dimension and infinite p-rank, where $p \geq 0$. Assume that every proper nonabelian subgroup of infinite p-rank of G has finite fundamental dimension. If G is not soluble and H is a normal subgroup of G having finite p-rank, then $H/T(H)$ is G-central.

Proof. Let $r_p(H) = r$. If $p = 0$ then we apply lemma 2.12 [11] and if $p > 0$ then we apply theorems A, E [7] to H and, in either case, we see that $H/T(H)$ is soluble and has finite special rank, which is a function of r. Let $n = r_0(H/T(H)$, which is dependent upon r only. Then H has series of G-invariant subgroups

$$T(H) = H_0 \leq H_1 \leq ... \leq H_d = H,$$

each of whose factors is abelian.

Note that $H_1/T(H)$ is torsion-free and of finite rank at most n so that $Aut(H_1/T(H)$ is isomorphic to a subgroup of $GL(n, \mathbf{Q})$. Hence $G/C_G(H_1/T(H))$ is a locally soluble group, which is isomorphic to a subgroup of $GL(n, \mathbf{Q})$. It follows from corollary 3.8 [10] that $G/C_G(H_1/T(H))$ is soluble and hence trivial by lemma 4. Thus $[G, H_1] \leq T(H)$.

Suppose inductively that $[G, H_{d-1}] \leq T(H)$. Then $H_{d-1}/T(H) \leq Z(G/T(H))$ so that $H/T(H)$ is nilpotent of class at most 2. Let $K/T(H) = Z(H/T(H))$. Then, as above, since $K/T(H)$ and H/K are torsion-free abelian and of rank at most n we have $[G, K] \leq T(H)$ and $[G, H] \leq K$. It follows by the three subgroups lemma and lemma 4 that $[G, H] = [G, G, H] \leq T(H)$ and the result now follows by induction on d. The lemma is proved.

Before proving our main theorem we need one more technical lemma.

Lemma 6. Let G be an insoluble, locally soluble subgroup of $GL(F, A)$ and let $p \geq 0$. Suppose that G has infinite fundamental dimension and infinite p-rank, $p \geq 0$. Assume that every proper nonabelian subgroup of infinite p-rank of G has finite fundamental dimension. Then G contains a proper normal subgroup V such that if U is a normal subgroup of G and $V \leq U \leq G$, $U \neq G$ then U is soluble and of finite fundamental dimension.

Proof. Let $T = T(G)$ and suppose at first that $T \neq G$ and $r_p(T)$ is finite (when $p = 0$ these conditions are automatically satisfied). By lemma 4 G/T is not soluble and hence is not simple by corollary 1 to theorem 5.27 [9]. Hence G contains a proper normal subgroup $L \geq T$, $L \neq T$. If $r_p(L)$ is finite lemma 5 implies that L/T is G-central and hence G/T contains a nontrivial maximal normal abelian subgroup V/T. Certainly $V \neq G$. If U is a normal subgroup of G and $V \leq U \leq G$, $V \neq U, U \neq G$ then $r_p(U)$ is infinite by lemma 5. From the choice of subgroup U it follows that U is nonabelian. By hypothesis U has finite fundamental dimension. The subgroup U is soluble by lemma 3. If there is no such subgroup L then we set $V = T$. Let U be a normal subgroup of G and $T \leq U \leq G$, $T \neq U, U \neq G$. The rank $r_p(U)$ is infinite and in the case of nonabelian subgroup U $ddim_F(U)$ is finite. If U is an abelian subgroup then there exists its nonabelian extension U_1, which is a proper subgroup. Therefore $ddim_F(U_1)$ is finite. Thus the fundamental dimension of U is finite.

Next suppose that $p > 0$. Suppose first that $r_p(T)$ is infinite. If a subgroup T is nonabelian and $T \neq G$ then T has finite fundamental dimension. In the case of abelian subgroup T, $T \neq G$, there exists its nonabelian extension K. Since a group G is insoluble then $K \neq G$. Therefore $ddim_F(K)$ is finite. Thus T has finite fundamental dimension. Then T is soluble by lemma 3. Therefore G/T is insoluble. Thus it is not simple by corollary 1 to theorem 5.27 [9]. If U is a normal subgroup of G and $T \leq U \leq G, U \neq G$ then $r_p(U)$ is infinite so U has finite fundamental dimension. By lemma 3 U is soluble. Thus we may set $T = V$.

Now let $T = G$. Suppose first that the Sylow p-subgroups of G are all of finite p-rank. Then G has the minimal condition on p-subgroups, by lemma 3.1 [12], and we obtain a contradiction, since in this case the p-subgroups are Chernikov and hence of finite, bounded ranks. Thus G contains some p-subgroup P of infinite rank. Hence G contains an infinite elementary abelian p-subgroup A, by corollary 2 to theorem 6.36 [9]. Certainly A is a proper subgroup of G then for every proper normal subgroup U of G we obtain that $UA \neq G$. Otherwise G/U is abelian, contrary to lemma 4. Thus UA is a proper subgroup of G of infinite p-rank. If subgroup UA is nonabelian then $ddim_F(UA)$ is finite. In the case of abelian subgroup UA there exists a proper nonabelian normal subgroup U_1 which contains U. Then $ddim_F(U_1A)$ is finite. Therefore $ddim_F(UA)$ is finite too. It follows that the fundamental dimension of U is finite and hence U is soluble. We set $V = 1$. The lemma is proved.

Now we turn to the main theorem on locally soluble groups. Let $d(G)$ be a solubility step of the group G.

Theorem 1. Let G be a nonabelian locally soluble subgroup of $GL(F, A)$ and $p \geq 0$. Suppose that G has infinite fundamental dimension and infinite p-rank. Assume that every proper nonabelian subgroup of infinite p-rank of G has finite fundamental dimension. Then G is a soluble group.

Proof. Suppose, on the contrary, that G is not soluble. By lemma 6 G contains a normal subgroup V with the property that if U is a normal subgroup of G and $V \leq U \leq G$, $U \neq G$ then U is soluble and of finite fundamental dimension. Set $V = U_0$, and let $d(U_0) = d_0$. Assume that we have constructed normal soluble subgroups $U_0 \leq U_1 \leq ... \leq U_n$ such that $d(U_i) = d_i$ for $i = 0, 1, ..., n$ and that $d_i < d_{i+1}$ for $i = 0, 1, ..., n-1$. Since G is not soluble, there exists a normal subgroup U_{n+1}, containing U_n, such that $d(U_{n+1}) = d_{n+1} > d(U_n)$ and we therefore obtain an ascending chain of soluble normal sobgroups of increasing derived lengths. Let $W = \bigcup_{n\geq 1} U_n$. By this construction W is not soluble and $V \leq W$, and we deduce that $W = G$.

Now let $A_n = [U_n, A]$ for each $n \in \mathbf{N}$. Since U_n is a normal subgroup of G, A_n is an FG-submodule for each n and since U_n has finite fundamental dimension then A_n is of finite dimensional. Thus $G/C_G(A/C_n)$ is a locally soluble finite dimensional linear group and hence is soluble, by corollary 3.8 [10]. We deduce from lemma 4 that $G = C_G(A/C_n)$, for each $n \in \mathbf{N}$. Since $G = \bigcup_{n\geq 1} U_n$ it follows that $C_A(G) = \bigcap_{n\geq 1} C_A(U_n) = \bigcap_{n\geq 1} C_n$ and hence G acts trivially on the factors of the series $0 \leq C_A(G) \leq A$. But in this case G is clearly abelian. This contradiction proves that G is actually soluble. The theorem is proved.

We recall that a group G has finite abelian section rank if every abelian section of G has finite p-rank for $p \geq 0$. We note that Baer and Heineken have shown that for soluble (and even hyperabelian) groups finite abelian section rank is equivalent to finite abelian subgroup rank (that is, the abelian subgroups of G have finite p-rank for $p \geq 0$). Futhermore, a group G has finite special rank $r(G) = r$ if every finitely generated subgroup of G can be generated by r elements and r is the least positive integer with this property. This notion is due to Mal'cev [13]. The special rank of a group is also called Prufer-Mal'cev rank. The following theorems are valid.

Theorem 2. Let G be a nonabelian locally soluble subgroup of $GL(F, A)$. Suppose that G has infinite fundamental dimension and infinite abelian section rank. Assume that every proper nonabelian subgroup of infinite abelian section rank of G has finite fundamental dimension. Then G is a soluble group.

Proof. Since G has infinite abelian section rank then there exists a prime p for which $r_p(G)$ is infinite. For this prime p every proper nonabelian subgroup H of infinite p-rank has infinite abelian section rank. Therefore H has finite fundamental dimension. Now we apply theorem 1. The theorem is proved.

Theorem 3. Let $G \leq GL(F, A)$ be a nonabelian group of infinite fundamental dimension and infinite special rank. Assume that every proper nonabelian subgroup of infinite special rank of G has finite fundamental dimension. If G is a locally soluble group then G is soluble.

Proof. Let G be a counterexample to the theorem and N be a proper normal subgroup of infinite special rank. If N is nonabelian than N has finite fundamental dimension. In the case of abelian subgroup N there exists a proper nonabelian normal subgroup N_1, which contains N. Thus $ddim_F(N_1)$ is finite. Therefore $ddim_F(N)$ is finite too. It follows from lemma 3 that N is soluble. If N is of finite special rank then by lemma 10.39 [9] N is hyperabelian. Let $\{N_\alpha\}$ be a set of all proper normal subgroups of G. Then $J = \prod N_\alpha$ is also hyperabelian. Since a simple locally soluble group is cyclic it follows that G is also hyperabelian. By theorem 7.1 [14] G contains a subgroup K that is either an elementary abelian q-subgroup of infinite special rank, for some prime q, or a torsion-free abelian subgroup of infinite special rank. Let N be a proper normal subgroup of G of finite special rank and let d be a natural number, guaranteed by lemma 10.39 [9], such that $N^{(d)}$ is a direct product of Chernikov p-groups, for different primes p. If $N^{(d)}K \neq G$ then it is easy to see that N is soluble. If $N^{(d)}K = G$ let r be some prime different from q and let X be the Sylow $\{q, r\}'$-subgroup of $N^{(d)}$. Then $XK \neq G$ and, as above, X, and hence N, is soluble. Besides NK is a proper subgroup of infinite special rank. If NK is nonabelian then $ddim_F(NK)$ is finite. Therefore $ddim_F(N)$ is finite too. In the case of abelian subgroup NK let N_1 be a maximal normal abelian subgroup of G, which contains N. It is possible to choose the proper nonabelian extension N_2 of subgroup N_1. Since $ddim_F(N_2)$ is finite then N has finite fundamental dimension. Thus, every proper normal subgroup of G is soluble and of finite fundamental dimension. The proof now proceeds as in the proof of theorem 1. The theorem is proved.

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